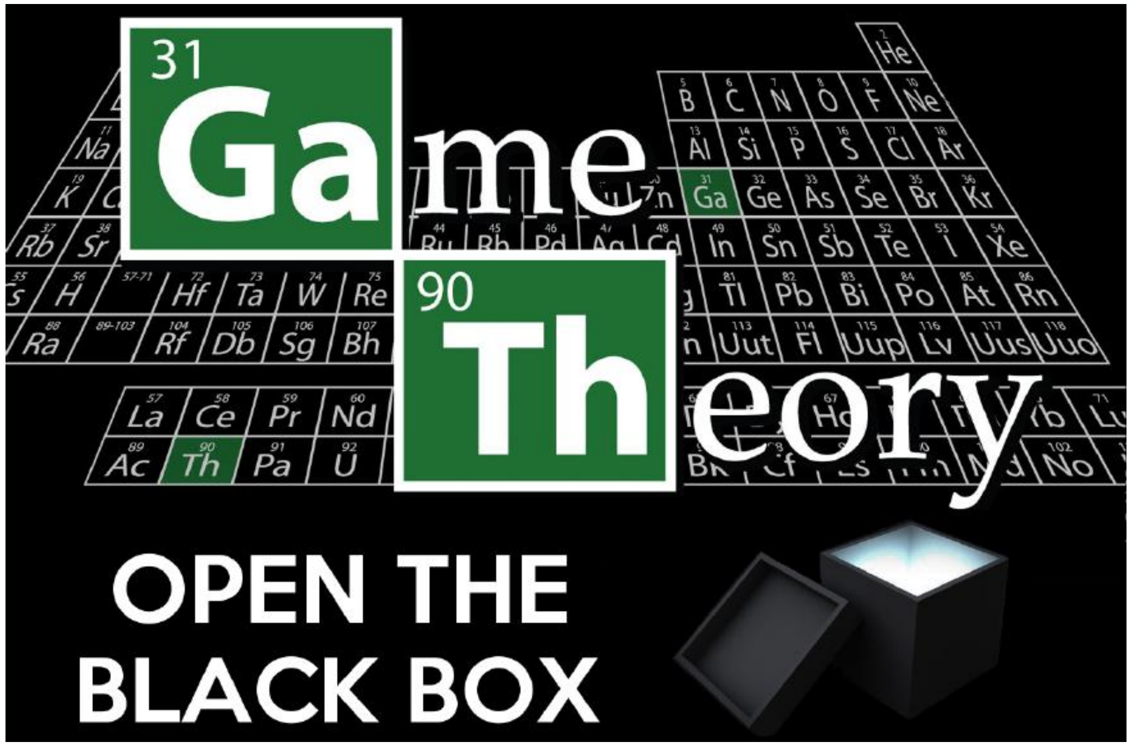
Chemical Game Theory



Jacob Kautzky
Group Meeting
February 26th, 2020

What is game theory?

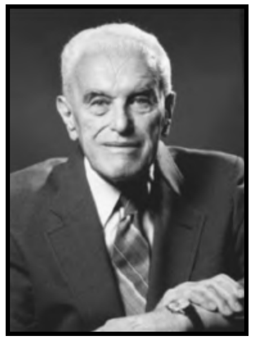
Game theory is the study of the ways in which interacting choices of rational agents produce outcomes with respect to the utilities of those agents

Why do we care about game theory?

11 nobel prizes in economics

John Nash

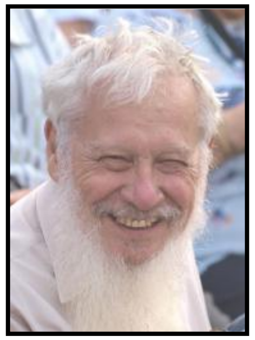
Reinhard Selten



John Harsanyi

1994 – "for their pioneering analysis of equilibria in the thoery of non-cooperative games"

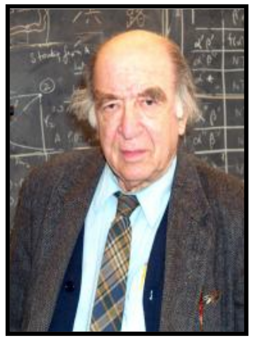
2005 – "for having enhanced our understanding of conflict and cooperation through game-theory"



Robert Aumann

Thomas Schelling

Why do we care about game theory?


Leonid Hurwicz

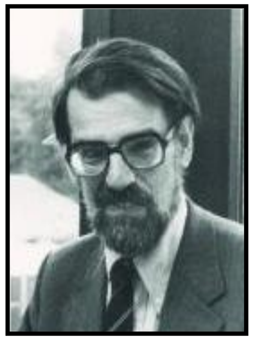

Eric Maskin


Roger Myerson

2007 – "for having laid the foundatiouns of mechanism design theory"

2012 – "for the theory of stable allocations and the practice of market design"


Alvin Roth

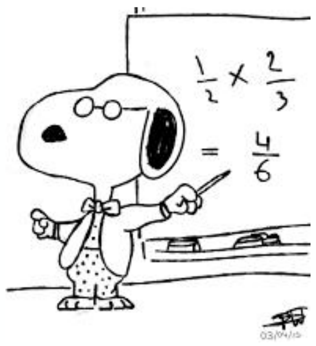

Lloyd Shapley


Jean Tirole

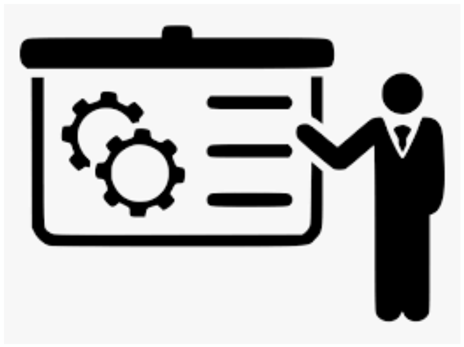
2014 – "for his analysis of market power and regulation"

Why do we care about game theory?

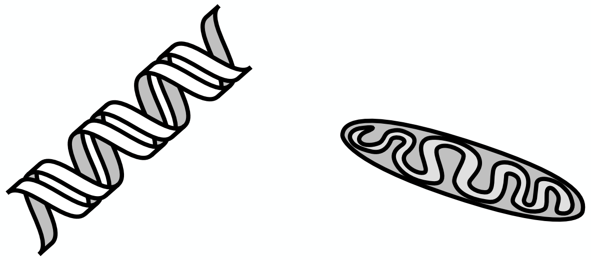
Mathematics



Business



Biology

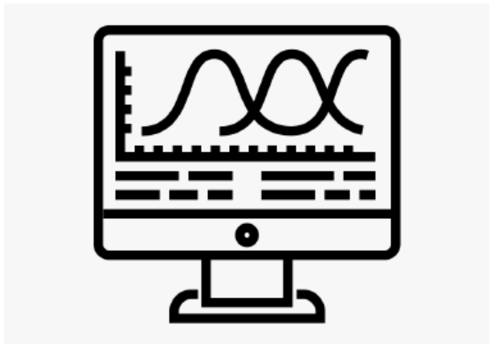


Engineering

Sociology

Philosophy

Computer Science

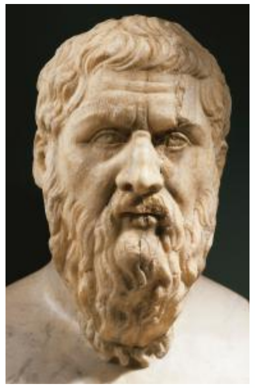


Political Science

Chemistry

Why do we care about game theory?

Plato, 5th Century BCE

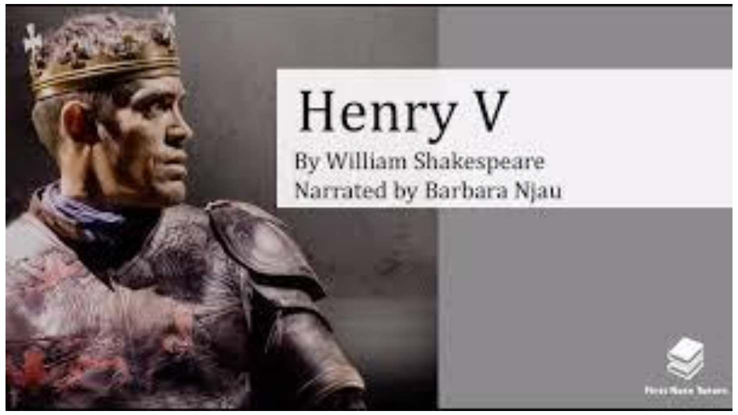


- Initial insights into game theory can be seen in Plato's work
- Theories on prisoner desertions

Cortez, 1517

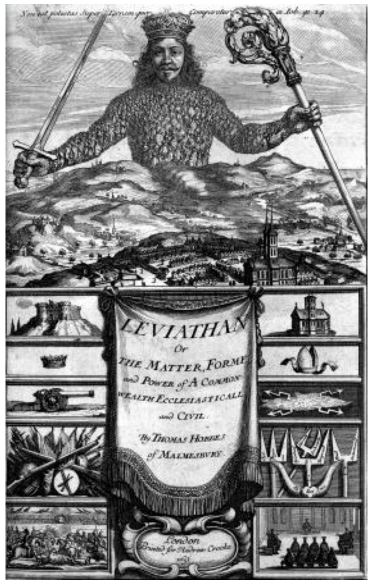
"burn the ships"

Shakespeare's Henry V, 1599



Henry orders the French prisoners executed infront of the French army

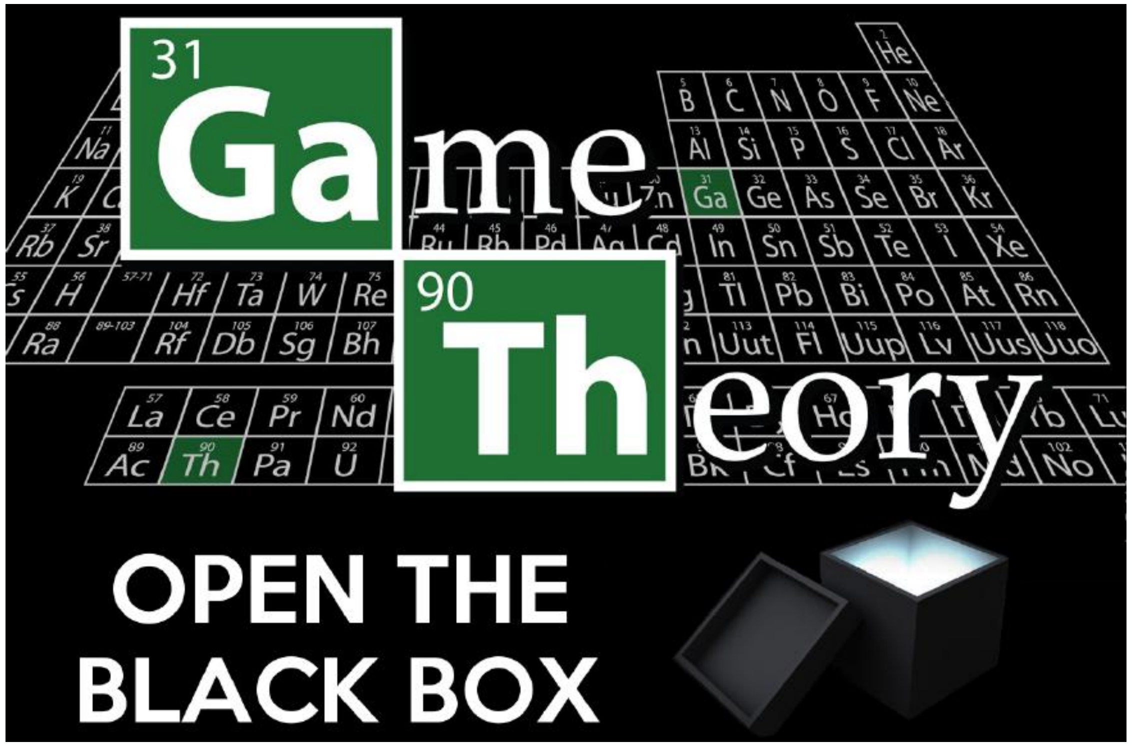
Hobbes' Leviathan, 1651



- **First mathematical theory of games was published in 1944 by John von Neumann and Oskar Morgenstern**

Chemical Game Theory

- **Basics of Game Theory**
    - Prisoners Dilemma
    - Battle of the Sexes
    - Rock Paper Scissors
    - Centipede Game
    - Iterated Prisoners Dilemma
- **Chemical Game Theory**
- **Game Theory in Computer Science**
- **Game Theory in Biology**
- **Game Theory in Chemistry**
    - Case 1: deciding an optimal dft functional
    - Case 2: inverse design



Game theory basics

Game theory analyzes the strategic interaction between at least 2 agents in their quest to achieve maximum utility

game – a set of cirumstances where the outcome is dependent on the actions of two or more decision makers

utility/ payoff – a quantification of the amount of use a player gets from a particular outcome

strategy – a complete plan of action a player will take given the set of circumstances that can arise within the game

Osborne, M. J. *An introduction to game theory; Oxford University Press: New York, NY, 2004.*

Game theory basics

simultaneous game

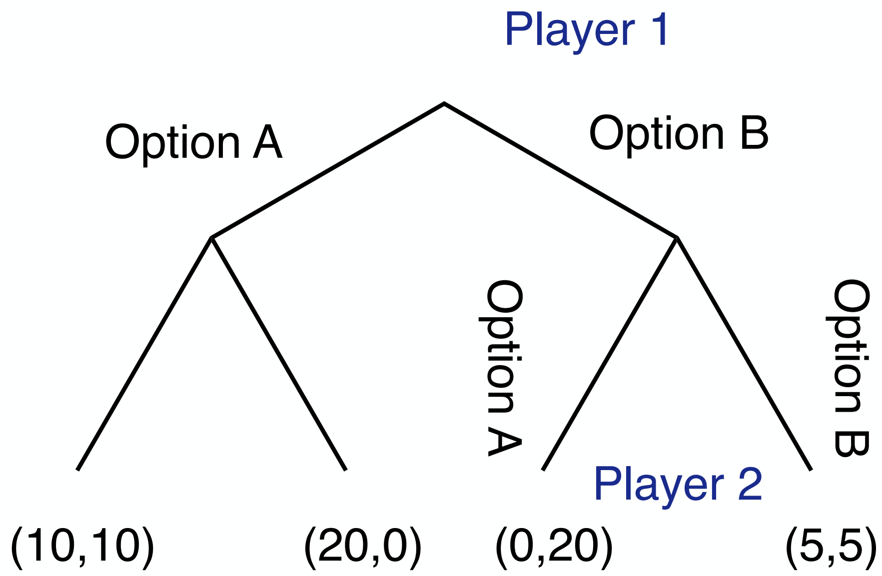
- players take their turns at the same time
- visualized as a matrix

	Option A	Option B
Option A	(10,10)	(0,20)
Option B	(20,0)	(5,5)

payoffs for both players listed in each box

sequential game

- players take their turns sequentially
- visualized as a directed graph



payoffs listed at the base of the tree

Osborne, M. J. *An introduction to game theory; Oxford University Press: New York, NY, 2004.*

Game theory basics

cooperative vs non-cooperative – whether players can estabilish alliances to maximize their winning chances

symmetric vs asymmetric – in a symmetric game, all players have the same overall goals, while in an asymmetic game participants have different or conflicting goals

perfect vs imperfect information – in perfect information all players can see other players moves, while in imperfect other player's moves are hidden

zero-sum vs non-zero sum games – in zero sum games, if a player gains something another player loses something while in non-zero sum games multiple players can gain at the same time

perfectly rational vs bounded rational – perfectly rational assumes all players are rational whereas bounded has individual player's rationality limited in some form

The scenarios discussed today will be primarily nocooperative, perfect information, and perfectly rational

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The prisoner’s dilemma

"So, ya believe in the Constitution, eh?"

“ I’ll give you a lighter sentence if you rat on your co-conspirator”

	Prisoner B tells	Prisoner B stays silent
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Not a stable state as B has a reason to snitch to get less jail time

Stable - a state where no player would change their move given the opportunity

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Telling is a dominant strategy for player A

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Nash Equilibrium (NE)

- An equilibration of entire sets of strategies
- Every finite game has at least one NE

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Battle of the sexes

A couple trying to decide between multiple options for a date night

		Partner B		
		Watch Football	Get a manicure	Go to a movie
Partner A	Watch Football	(20,10)	(7,7)	(7,5)
	Get a manicure	(0,0)	(5,20)	(0,5)
	Go to a movie	(5,0)	(5, 7)	(20,20)

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Battle of the sexes

There can be multiple Nash Equilibria

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	Go to a movie	(5,0)	(5, 7)	(20,20)

Pareto Optimum - an outcome where there is no other outcome where every other player is at least as well off

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Rock paper scissors

Player 2

Player 1

	Rock	Paper	Scissors
Rock	(0,0)	(-1,1)	(1,-1)
Paper	(1,-1)	(0,0)	(-1,1)
Scissors	(-1,1)	(1, -1)	(0,0)

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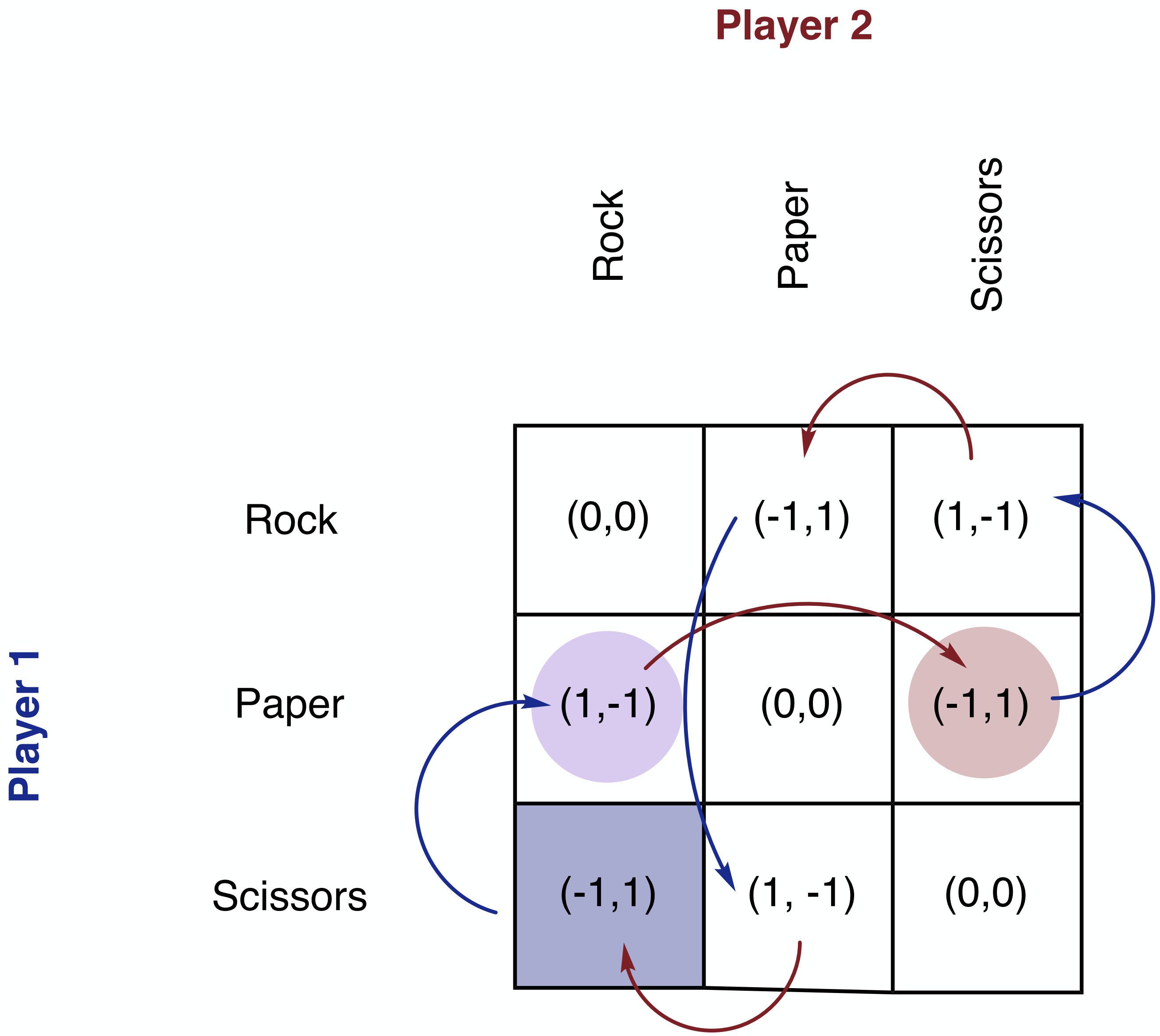
Rock paper scissors



Player 2

Player 1

	Rock	Paper	Scissors
Rock	(0,0)	(-1,1)	(1,-1)
Paper	(1,-1)	(0,0)	(-1,1)
Scissors	(-1,1)	(1, -1)	(0,0)

Pure Strategy - a player chooses one option 100% of the time

Mixed Strategy - a player chooses multiple options with differing probabilities

Osborne, M. J. *An introduction to game theory; Oxford University Press: New York, NY, 2004.*

Rock paper scissors

Player 2

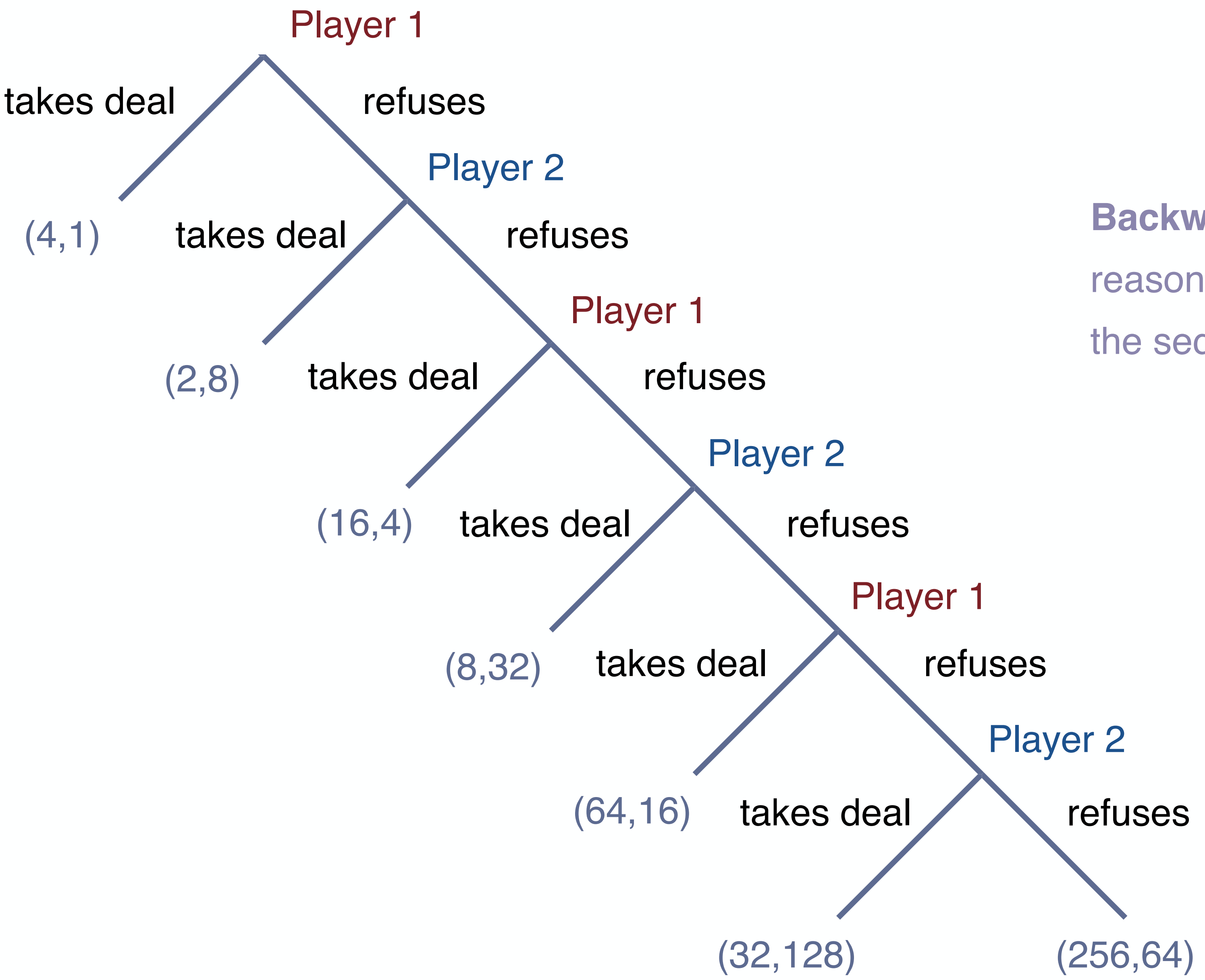
			Rock	Paper	Scissors
			1/3	**1/3**	**1/3**
Player 1	Rock	**1/3**	(0,0)	(-1,1)	(1,-1)
	Paper	**1/3**	(1,-1)	(0,0)	(-1,1)
	Scissors	**1/3**	(-1,1)	(1, -1)	(0,0)

Player 1's Expected Utility : 1/9 * 0 + 1/9 * -1 + 1/9 * 1 + 1/9 * 1 + 1/9 * 0 + 1/9 * -1 + 1/9 * -1 + 1/9 * 1 + 1/9 * 0 = 0

Osborne, M. J. *An introduction to game theory; Oxford University Press: New York, NY, 2004.*

Nash equilibria in sequential games

The Centipede Game – *A game played by two players where starting with $5 each player can either accept the deal and get 4/5 of the pot or pass the deal at which point the money in the pot doubles and the same offer is made to the other player until the pot reaches a grand total of $320 dollars*



Player 1
takes deal
refuses
(4,1)
Player 2
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refuses
(2,8)
Player 1
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refuses
(16,4)
Player 2
takes deal
refuses
(8,32)
Player 1
takes deal
refuses
(64,16)
Player 2
takes deal
refuses
(32,128)
(256,64)

Backward Induction - the process of reasoning backward in time to determine the sequence of optimal events

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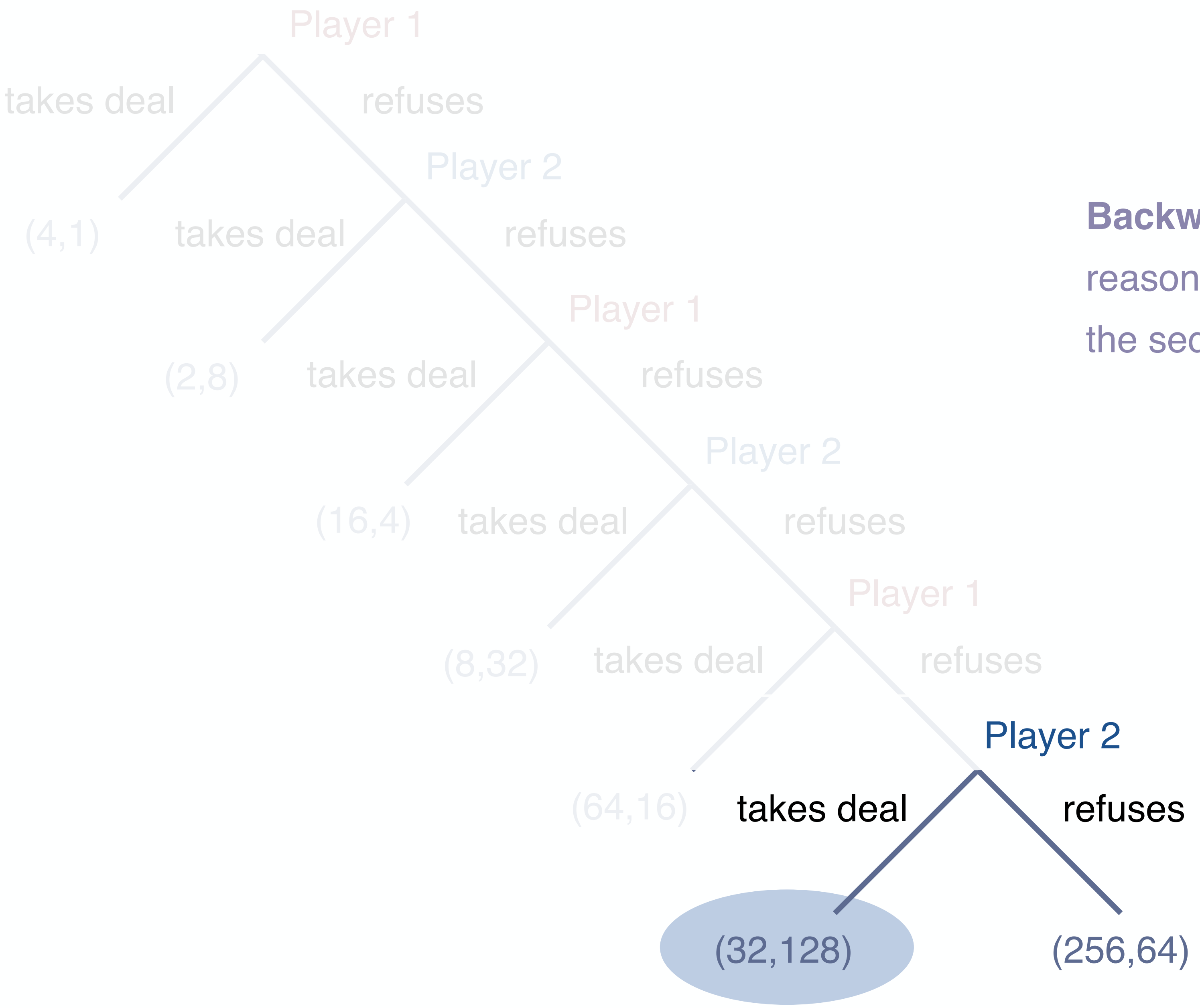

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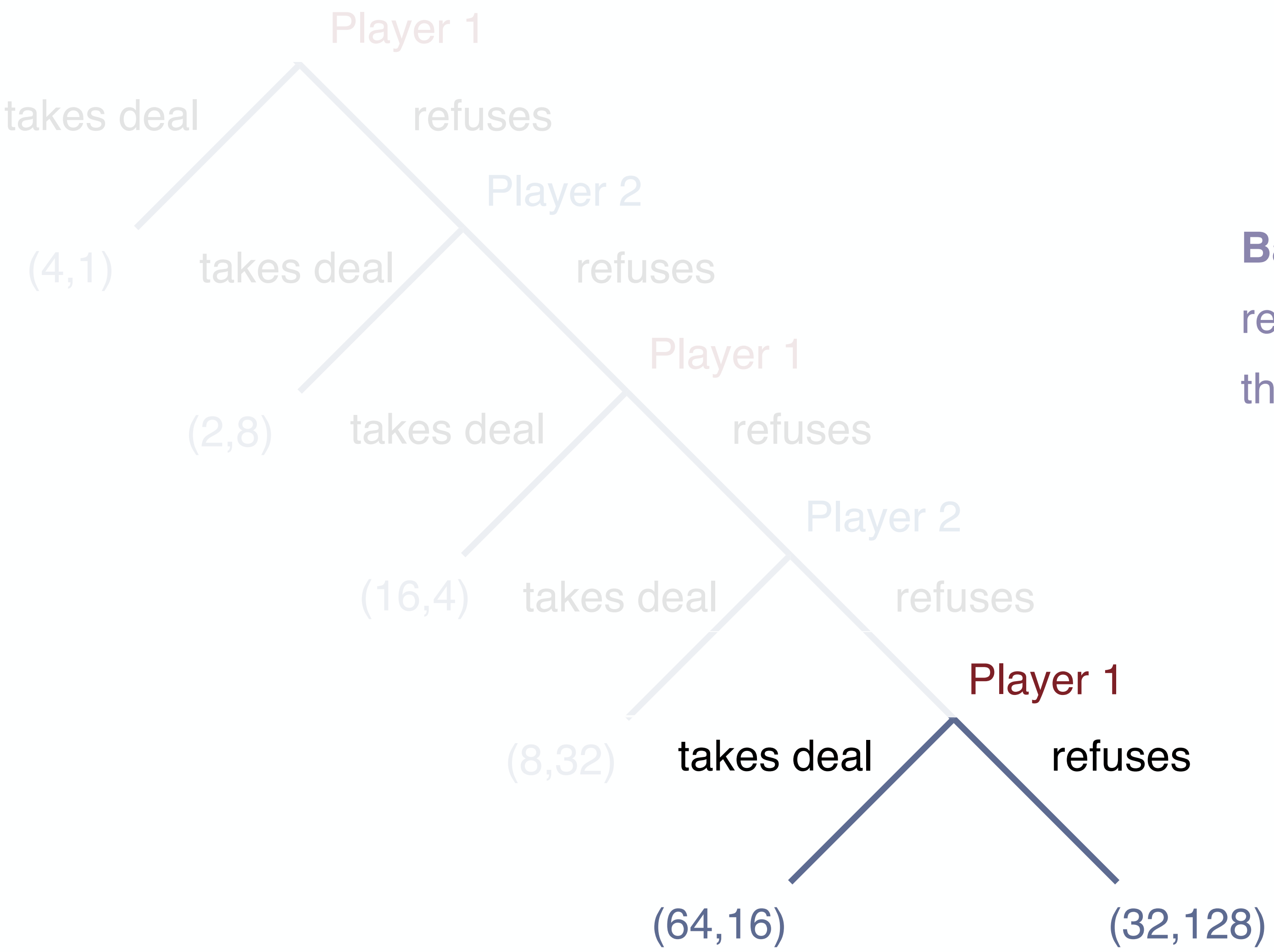

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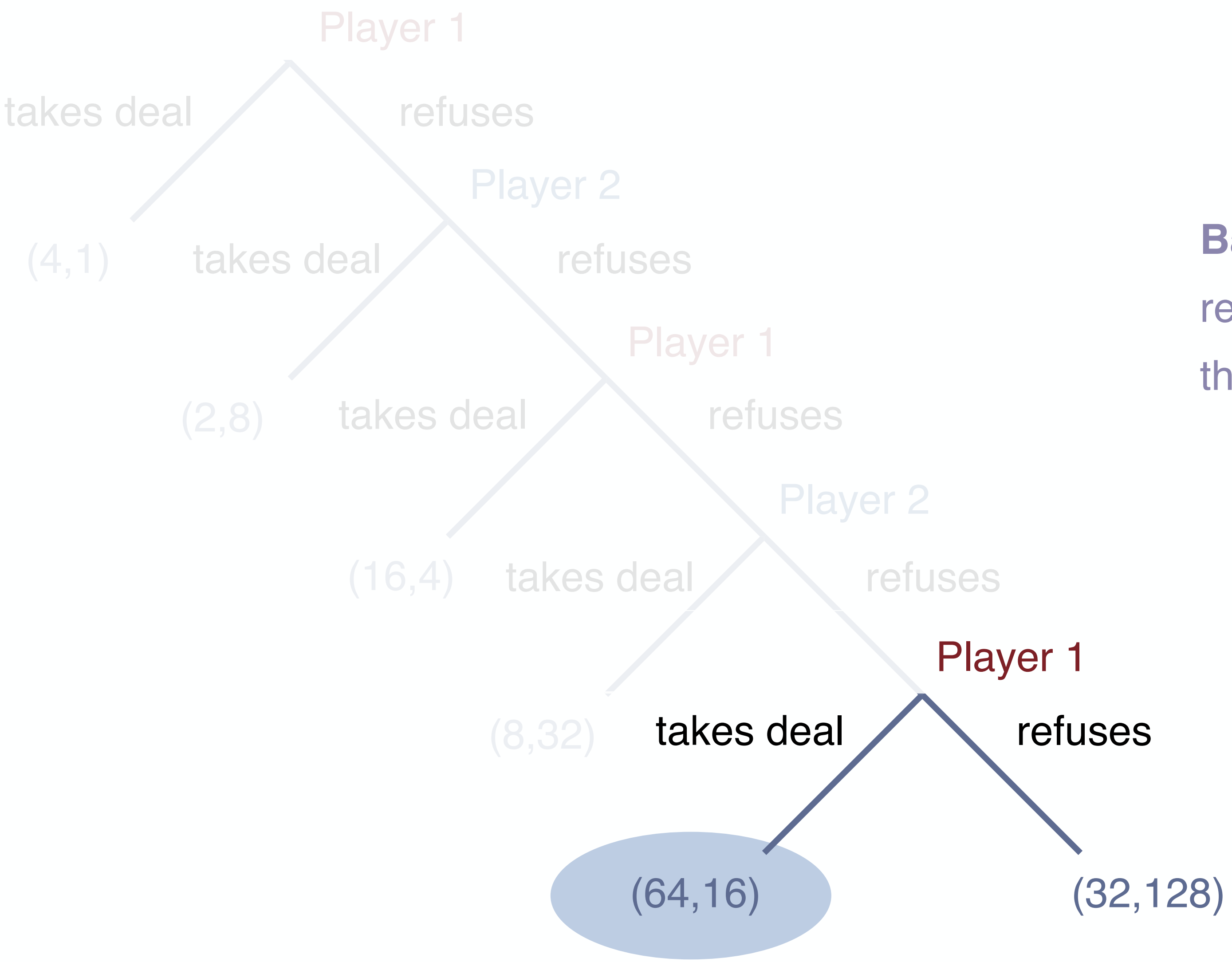

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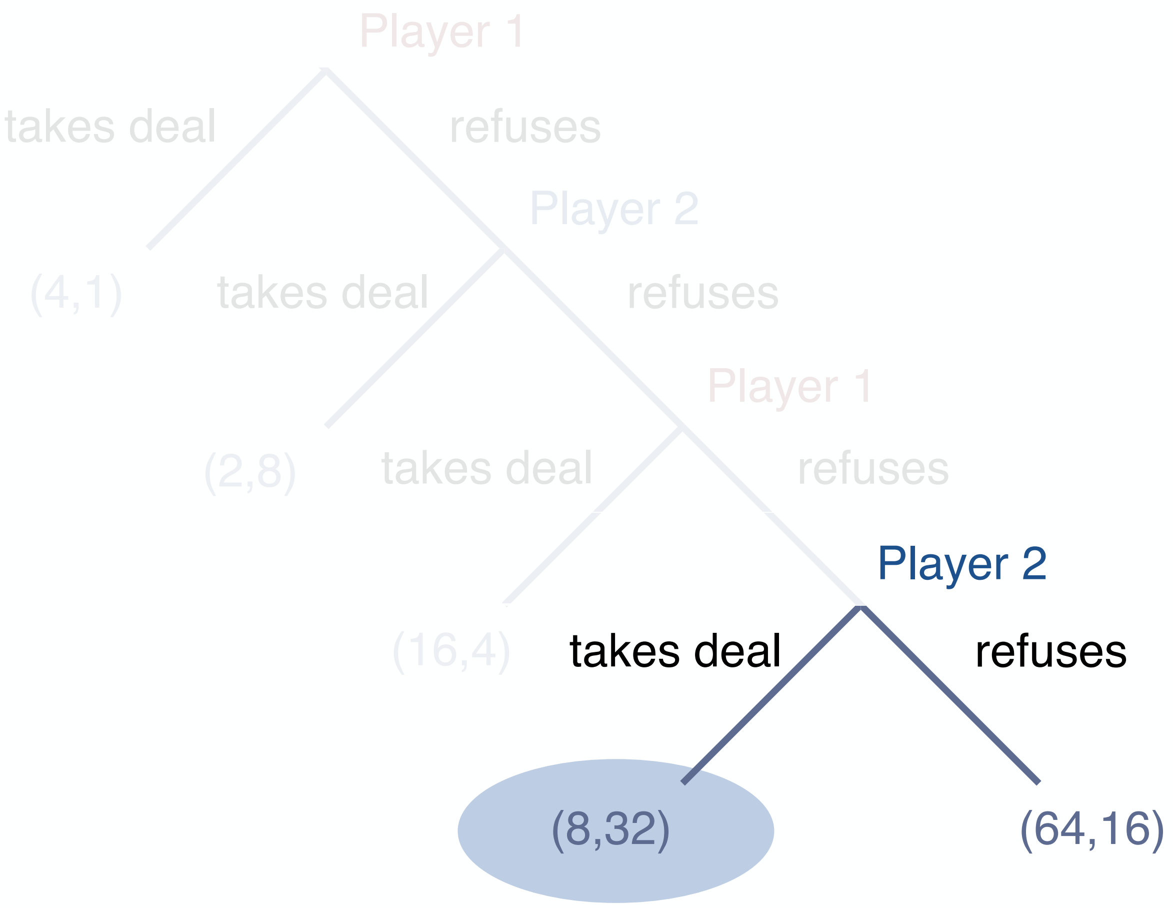

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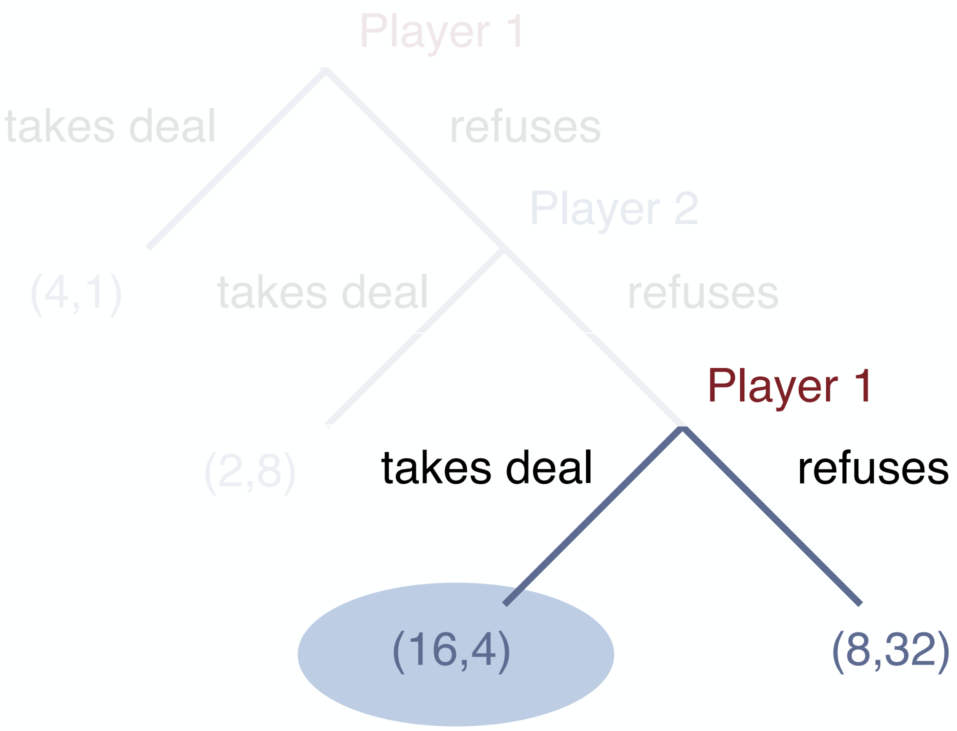

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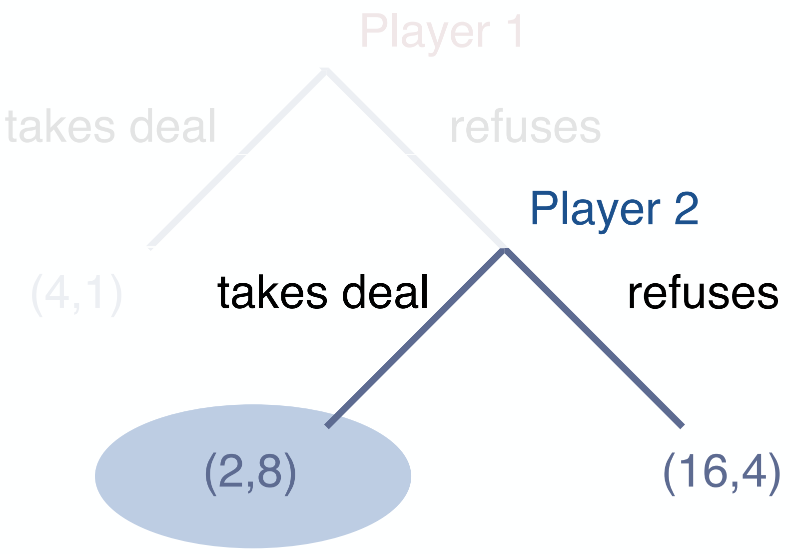

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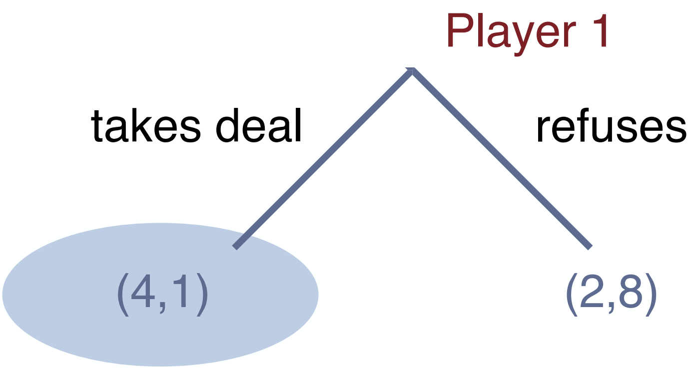

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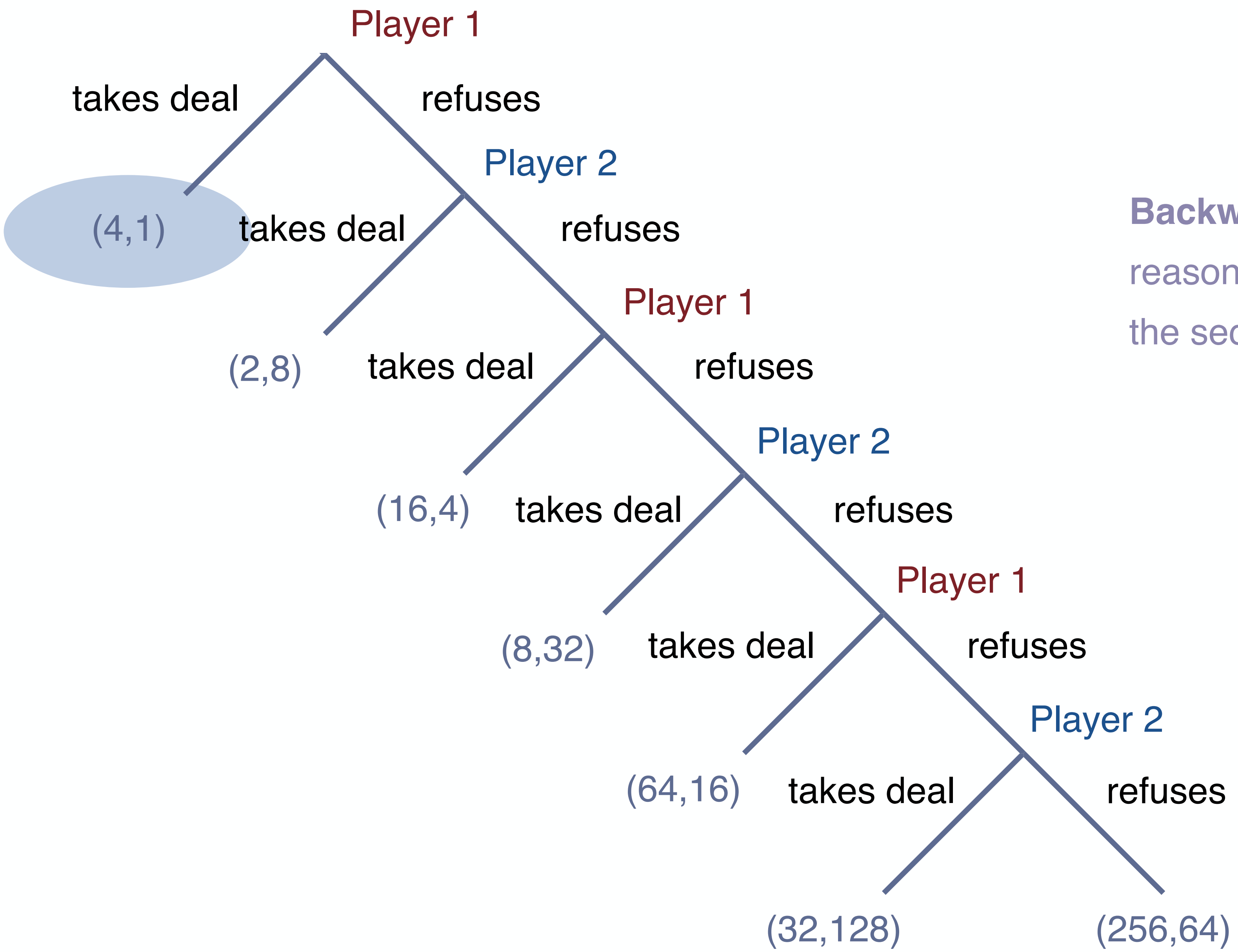

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Backward Induction - the process of reasoning backward in time to determine the sequence of optimal events

NE is for player 1 to take the first deal!

What happens when we move away from finite games?

Iterated Prisoners Dilemma

	Prisoner B tells	Prisoner B stays silent
Prisoner A tells	(10,10)	(0,20)
Prisoner A stays silent	(20,0)	(5,5)

- Repeat the prisoners dilemma over and over again
  - Players can learn about the behavioral tendencies of their opponents

In the early 1980’s Robert Axelrod had a tournament where users submitted different algorithms for the iterated prisoners dilemna

Osborne, M. J. *An introduction to game theory; Oxford University Press: New York, NY, 2004.*

Iterated Prisoners Dilemma

Unconditional Cooperator – always cooperates regardless of what the opponent does

Unconditional Defector – always defects regardless of what the opponent does

Random – player defects with a given probability p

GRIM/ TRIGGER – cooperates until their opponent defects once, at which point it switches to unconditional defection

Tit for Tat – cooperates on the first round and immitates their opponents move thereafter

⋮

and a range of others as well

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Random – player defects with a given probability p

GRIM/ TRIGGER – cooperates until their opponent defects once, at which point it switches to unconditional defection

Tit for Tat – cooperates on the first round and immitates their opponents move thereafter

Win-stay Lose-shift – cooperates if it and its opponent moved the same in the previous move and defects otherwise

Gradual Tit for Tat – tit for tat, but (1) it gradually increases the number of defections for each additional defection of its opponent and (2) it cooperates the next 2 rounds after it defects

Osborne, M. J. *An introduction to game theory; Oxford University Press: New York, NY, 2004.*

Chemical Game Theory

Basics of Game Theory

- Prisoners Dilemma
- Battle of the Sexes
- Rock Paper Scissors
- Centipede Game
- Iterated Prisoners Dilemma

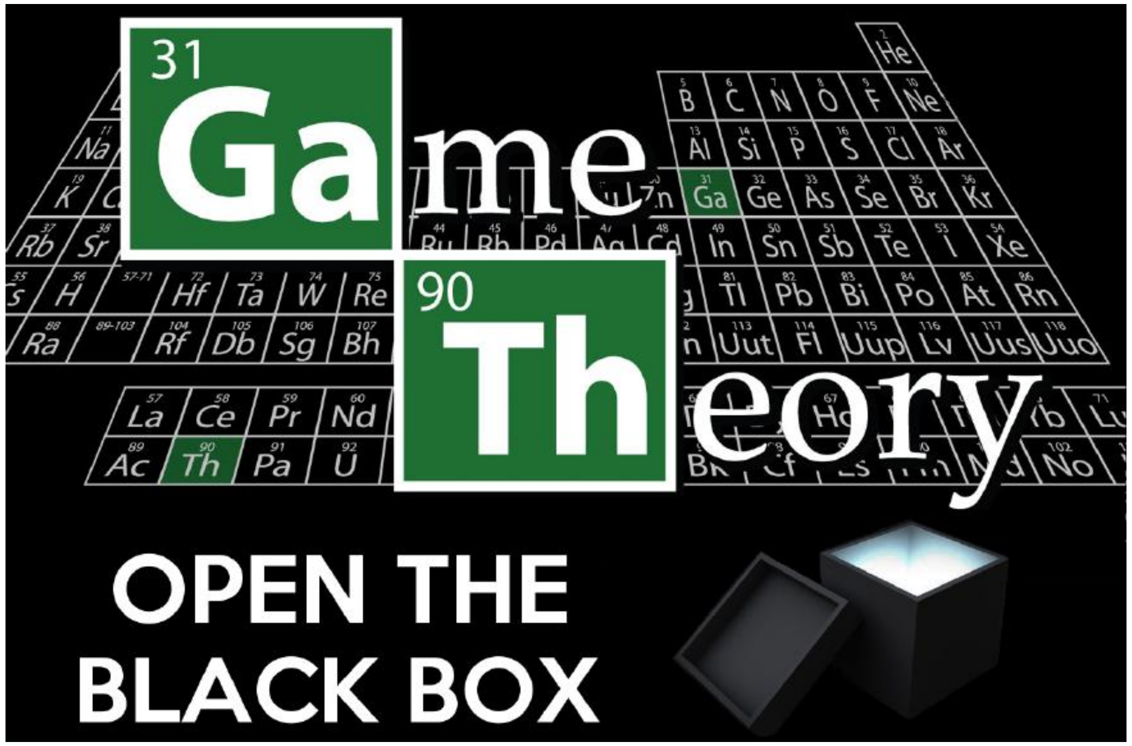
Chemical Game Theory

Game Theory in Computer Science

Game Theory in Biology

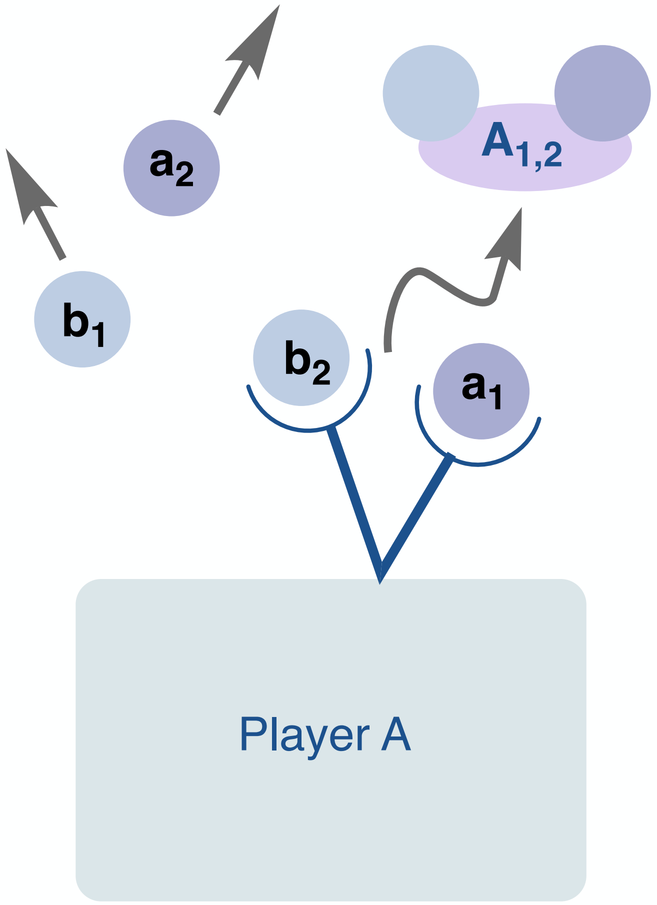
Game Theory in Chemistry

- Case 1: deciding an optimal dft functional
- Case 2: inverse design



Chemical Game Theory (CGT)

- Predictive rather than normative
- Takes into account players biases, altruism, deception, imperfect information, and relative pain levels



*Considers the player's strategies as "**knowlecules**"*

*CGT is concerned with **decision reactions** between the players and their choices form decisions*

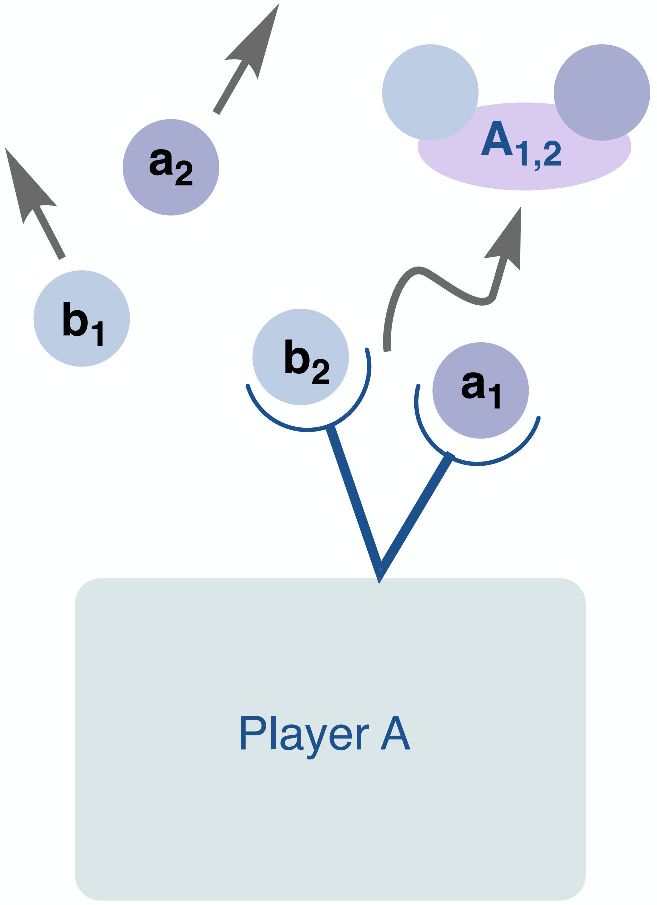
Each player must consider how the other player "reactors" will act and how subsequent reactors will respond

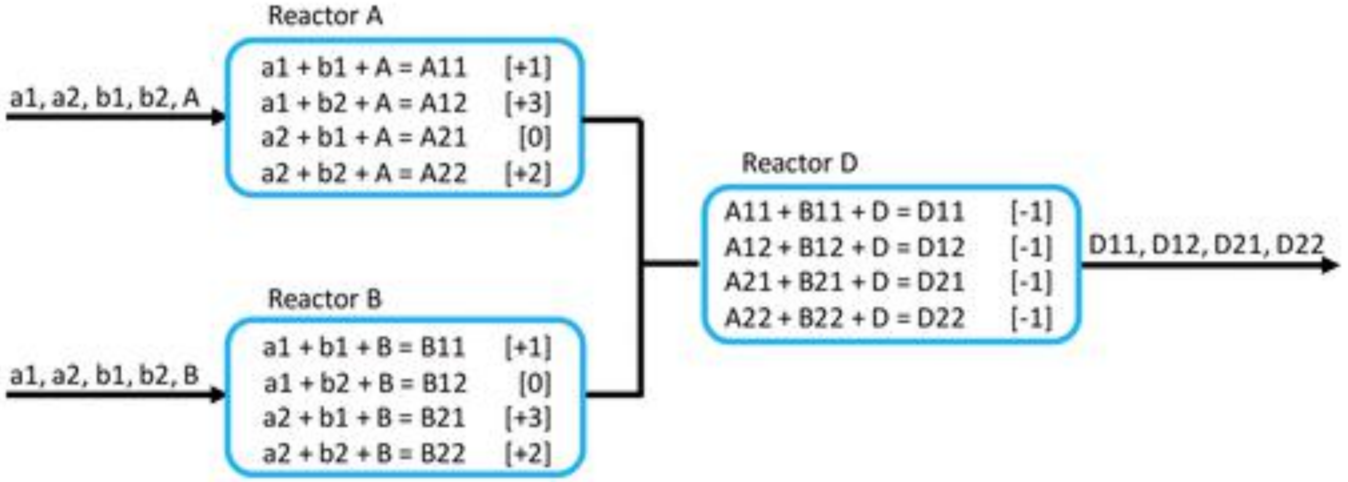
Each reaction has an energy of reaction related to the amount of pain or utility given to that choice

The system then searches for a form of chemical equilibria

Velegol, D.; Suhey, P. Connolly, J.; Morrissey, N.; Cook, L. *Ind. Eng. Chem. Res.* **2018**, *57*, 13593.

Chemical Game Theory (CGT) applied to the prisoners dilemma





Reactor A

a1, a2, b1, b2, A

a1 + b1 + A = A11 [+1]
a1 + b2 + A = A12 [+3]
a2 + b1 + A = A21 [0]
a2 + b2 + A = A22 [+2]

Reactor B

a1, a2, b1, b2, B

a1 + b1 + B = B11 [+1]
a1 + b2 + B = B12 [0]
a2 + b1 + B = B21 [+3]
a2 + b2 + B = B22 [+2]

Reactor D

A11 + B11 + D = D11 [-1]
A12 + B12 + D = D12 [-1]
A21 + B21 + D = D21 [-1]
A22 + B22 + D = D22 [-1]

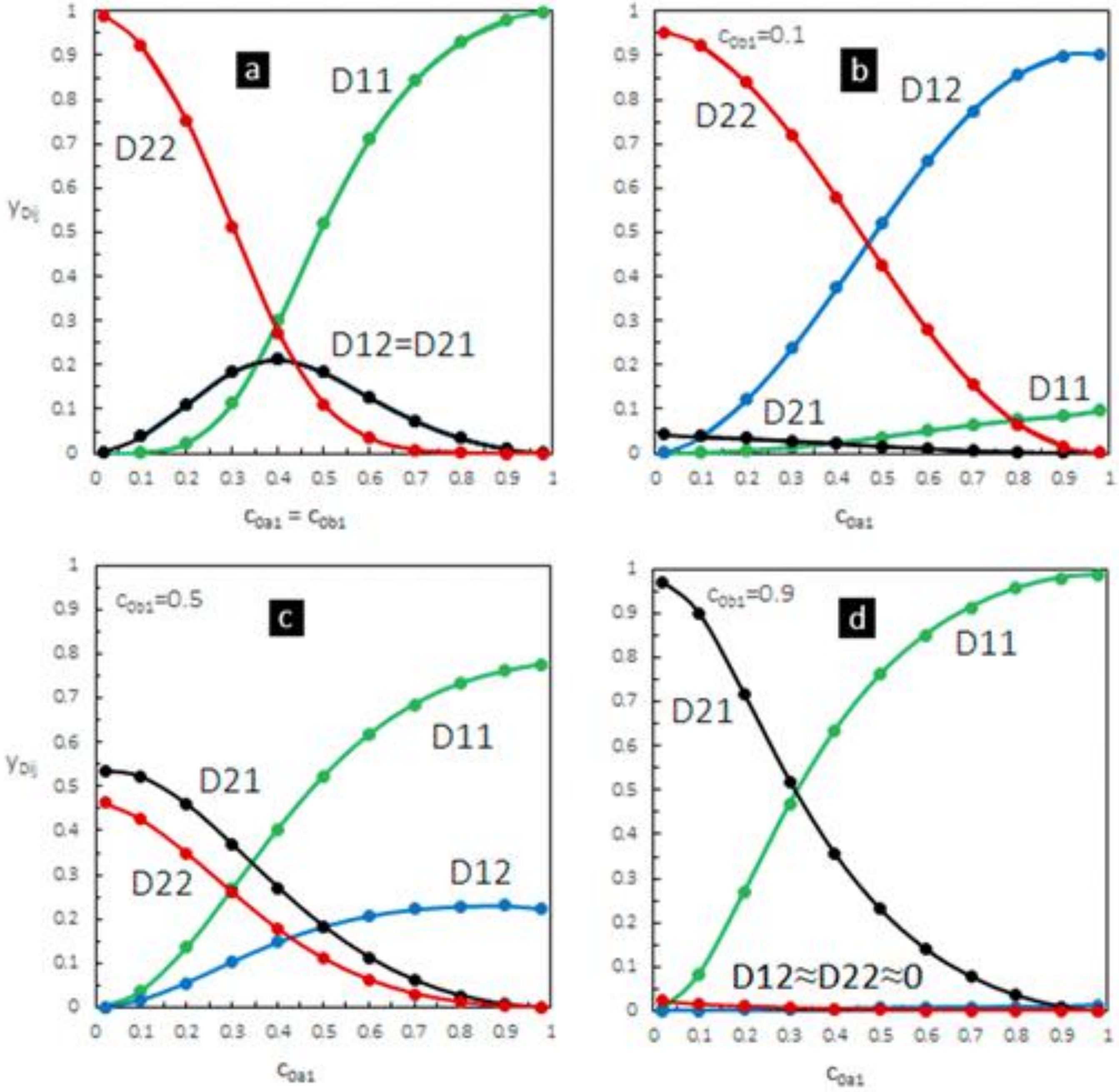
D11, D12, D21, D22

Treat each player as a reactor as well as a reactor for the decider

	b_1 = quiet	b_2 = tell
a_1 = quiet	(1,1)	(3,0)
a_2 = tell	(0,3)	(2,2)

Velegol, D.; Suhey, P. Connolly, J.; Morrissey, N.; Cook, L. *Ind. Eng. Chem. Res.* **2018**, *57*, 13593.

Chemical Game Theory (CGT) applied to the prisoners dilemma



depending on the different parameters selected you get all 4 outcomes as opposed to just the tell–tell for the NE

Velegol, D.; Suhey, P. Connolly, J.; Morrissey, N.; Cook, L. *Ind. Eng. Chem. Res.* **2018**, *57*, 13593.

Chemical Game Theory

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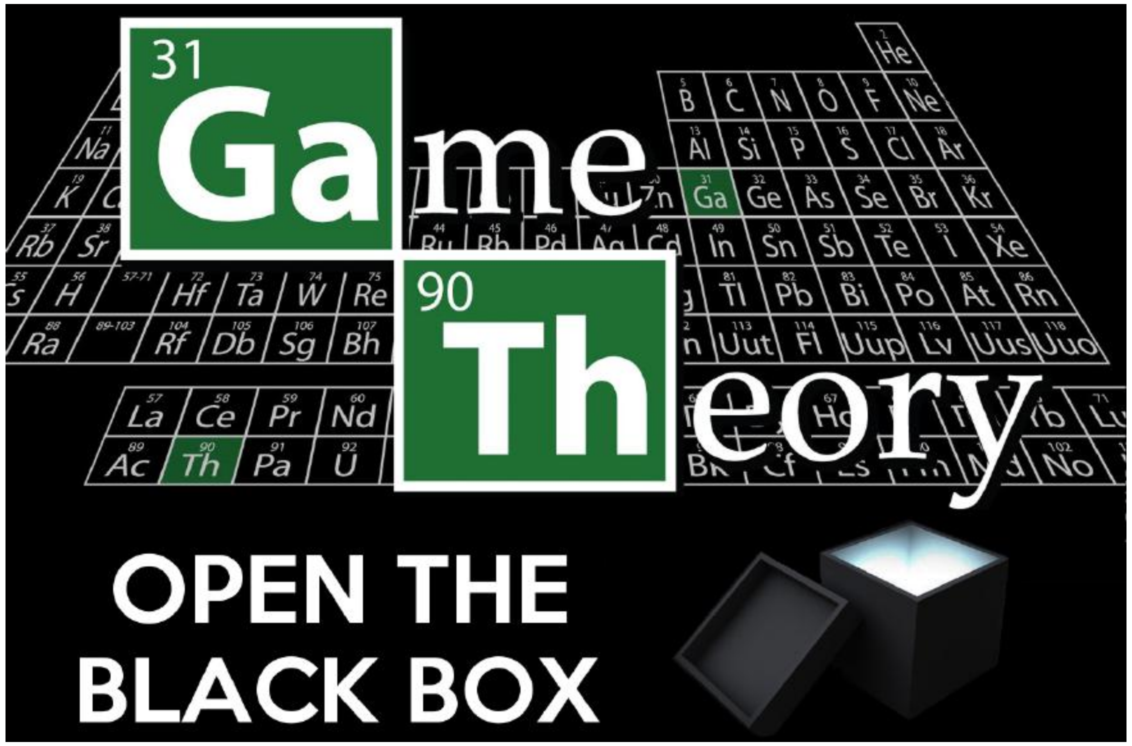
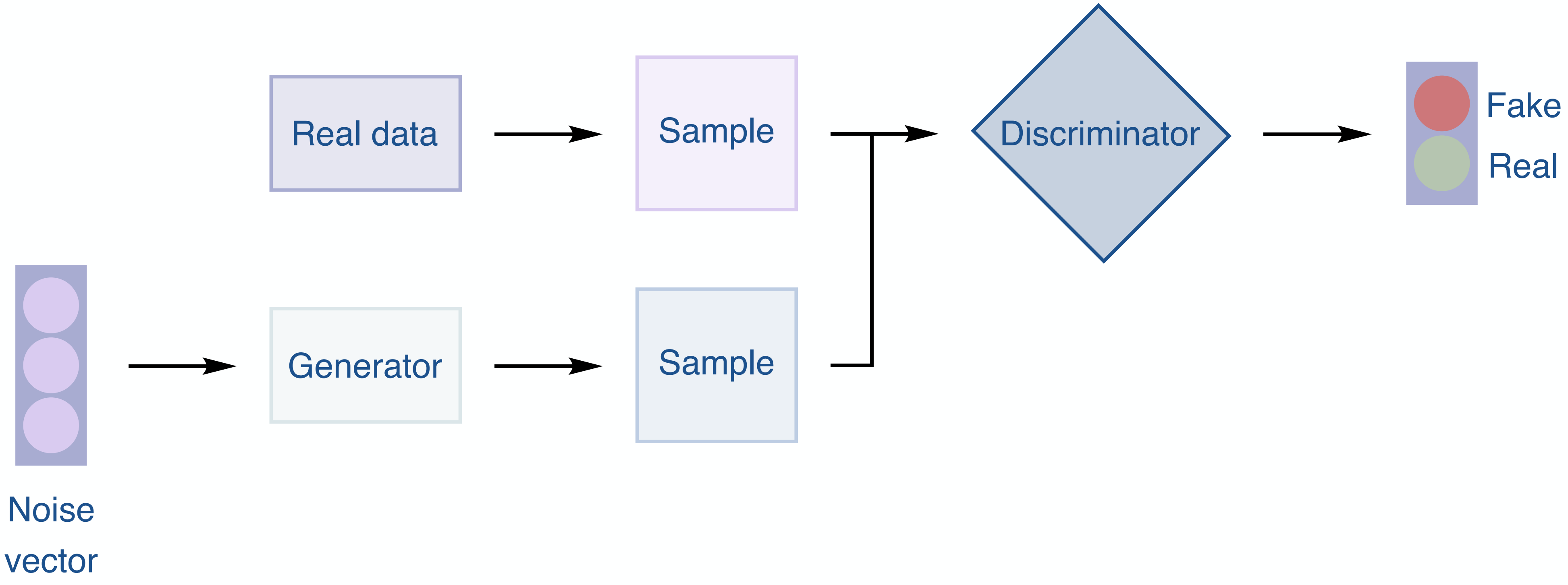


General Adversarial Networks (GANs)

- Consists of a generator and discriminator
- The generator is a form of unsupervised learning and it takes numbers random numbers and returns a sample
- This sample as well as a sample pulled from real data are then put into a discriminator
- A discriminator is a form of supervised learning that tries to determine if the data is real or fake
- This data is then returned to the generator and the process is iterated



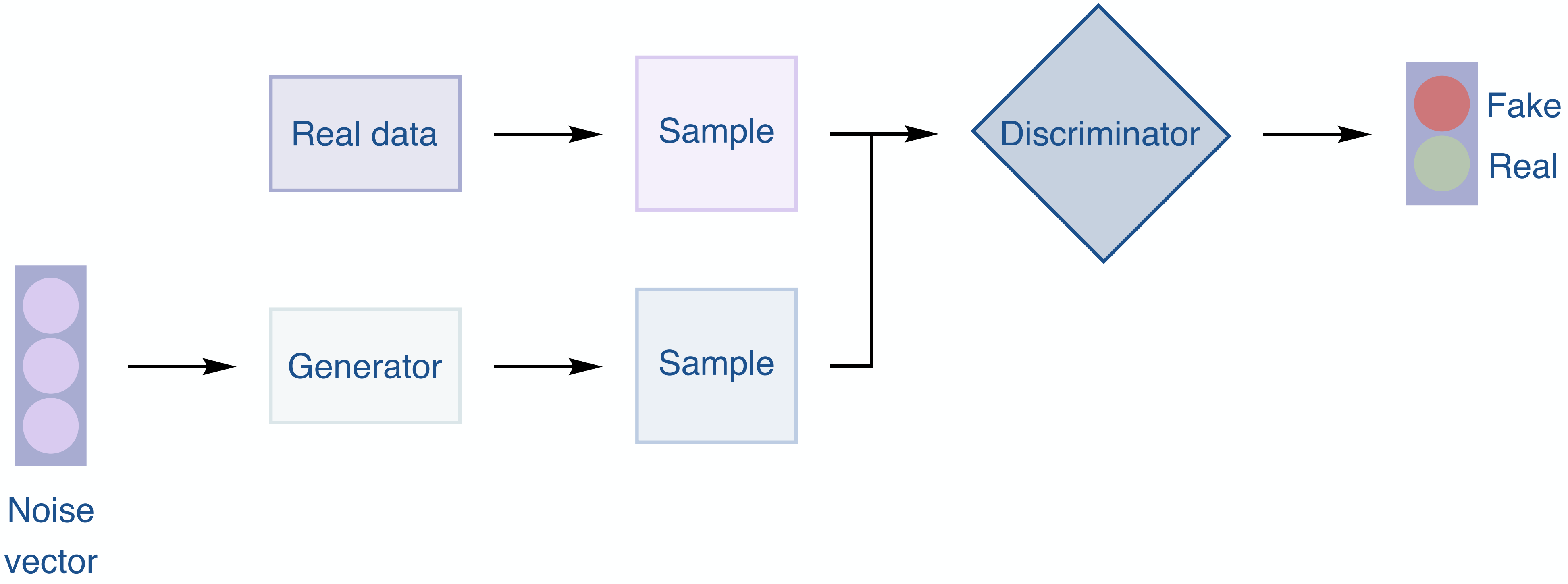
Bell, J. *Machine Learning: Hands-On for Developers and Technical Professionals; John Wiley & Sons, Inc.: Indianopolis, IN, 2014.*

General Adversarial Networks (GANs)

Viewed as a form of inverse game theory

Inverse game theory aims to design a game based on a players strategies and aims

Inverse game theory plays an important role in developing AI agent environments



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General Adversarial Networks (GANs)

“[GANs] are the most interesting idea in the last 10 years in ML” – Facebook’s AI research director Yann Lecun

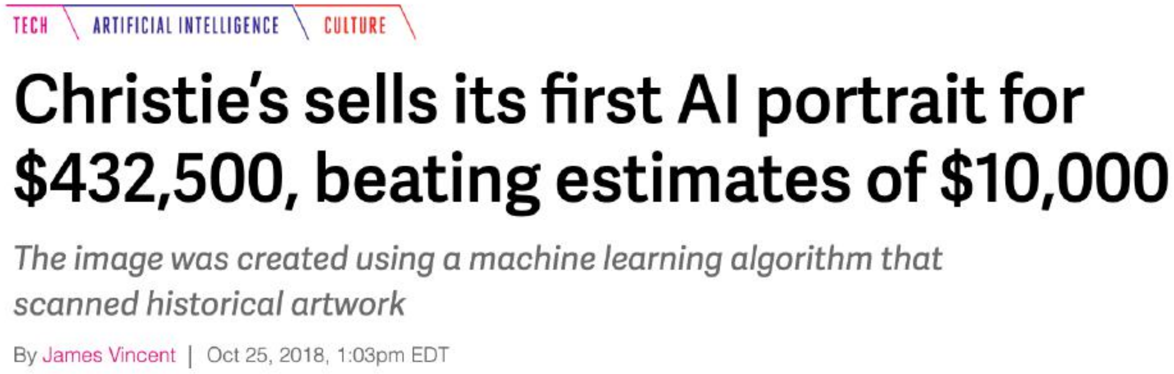
faces generated from a GAN

Bell, J. *Machine Learning: Hands-On for Developers and Technical Professionals; John Wiley & Sons, Inc.: Indianapolis, IN, 2014.*

General Adversarial Networks (GANs)

“[GANs] are the most interesting idea in the last 10 years in ML” – Facebook’s AI research director Yann Lecun

Trained a GAN by feeding it historical paintings



TECH ARTIFICIAL INTELLIGENCE CULTURE

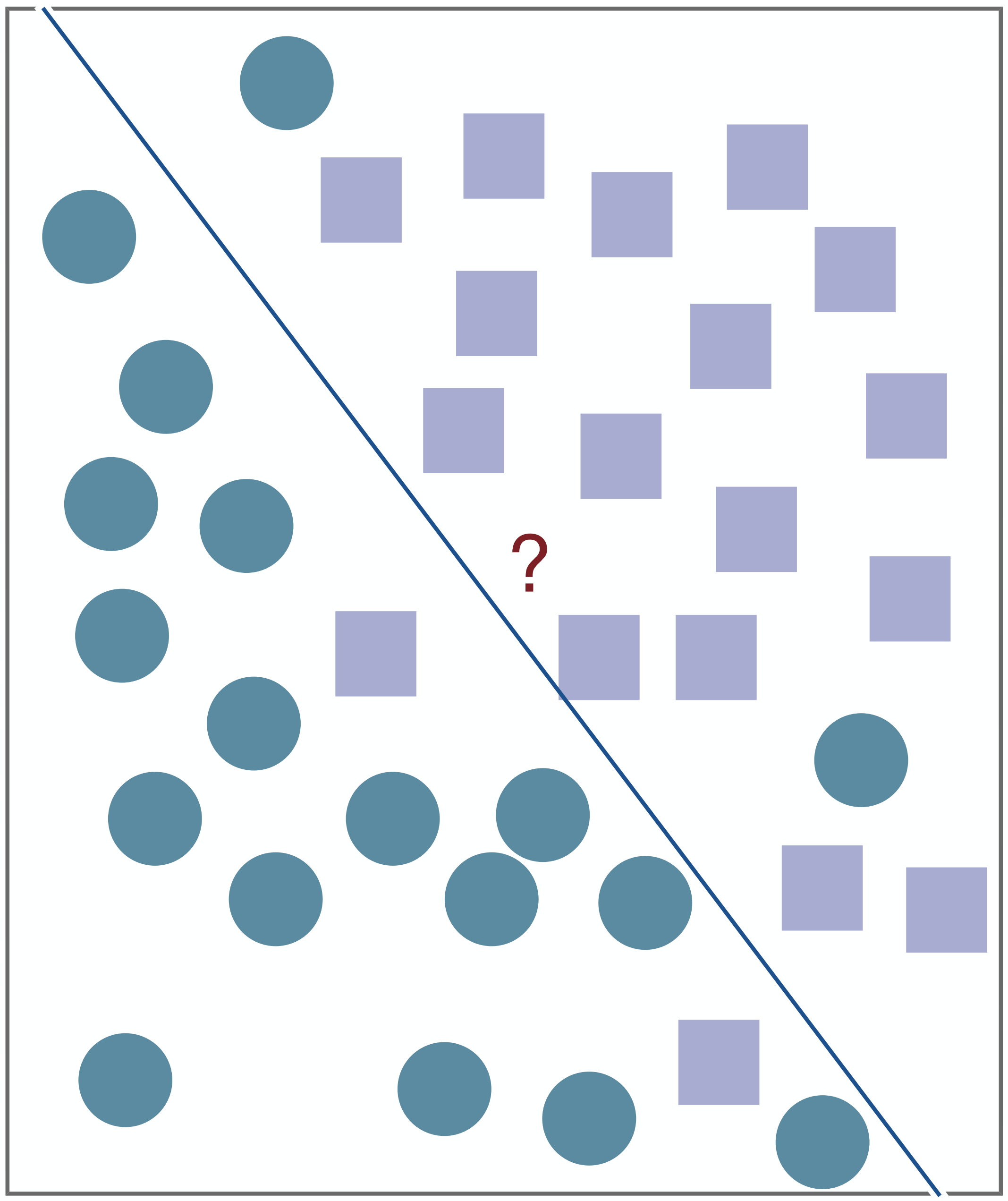
Christie’s sells its first AI portrait for $432,500, beating estimates of $10,000

The image was created using a machine learning algorithm that scanned historical artwork

By James Vincent | Oct 25, 2018, 1:03pm EDT

Bell, J. *Machine Learning: Hands-On for Developers and Technical Professionals; John Wiley & Sons, Inc.: Indianapolis, IN, 2014.*

Support Vector Machines (SVM)

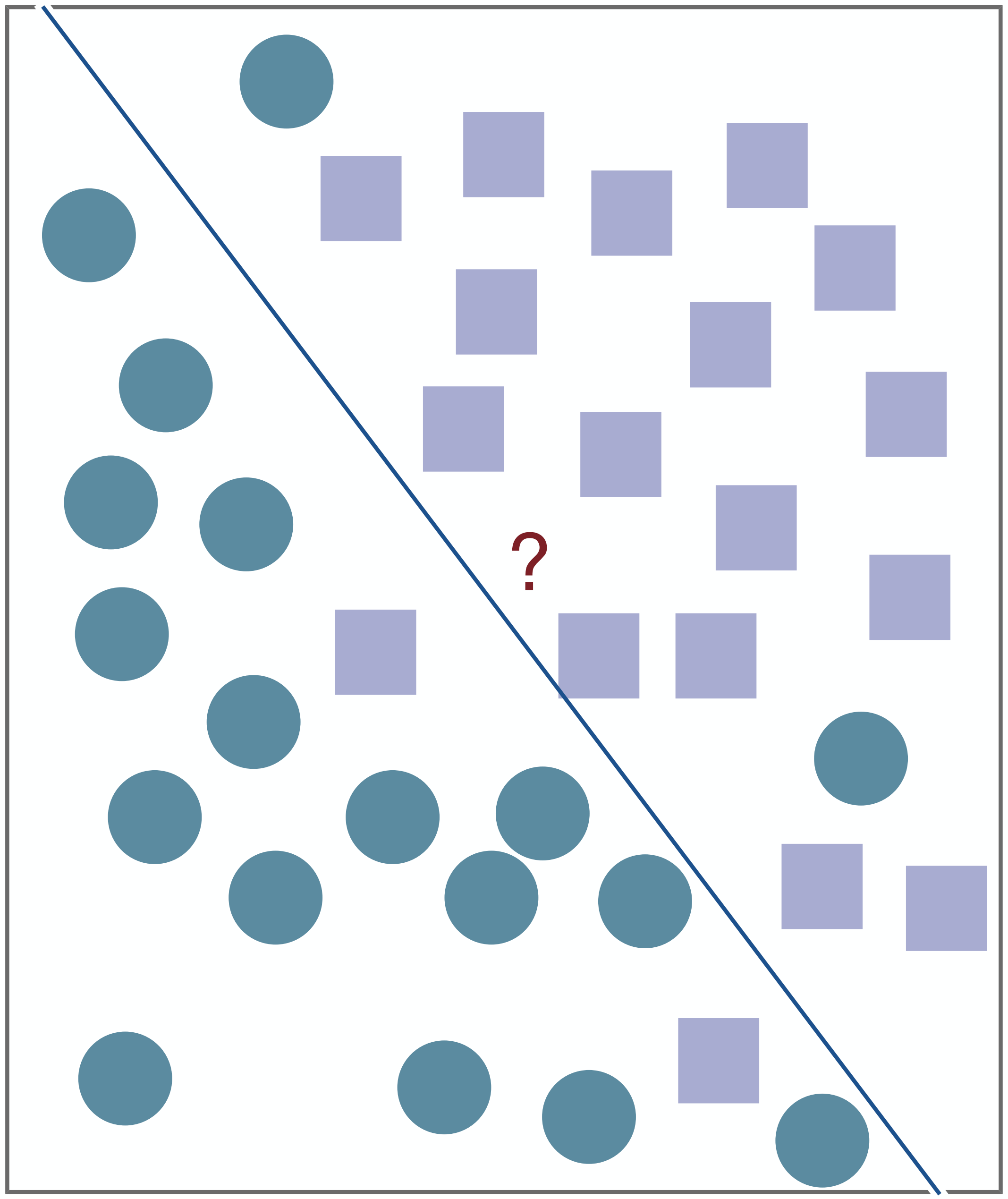


? would be assigned ■

- Classifying algorithm
- Supervised learning
- The algorithm searches for a decision boundary or separating hyperplane that leads to the best separation
- Quickly trained, works well for high-dimensional data, relatively good at not overfitting, not very interpretable
- Commonly used method; used by Doyle and Cronin amongst others

Bell, J. *Machine Learning: Hands-On for Developers and Technical Professionals; John Wiley & Sons, Inc.: Indianopolis, IN, 2014.*

Support Vector Machines (SVM)



? would be assigned ■

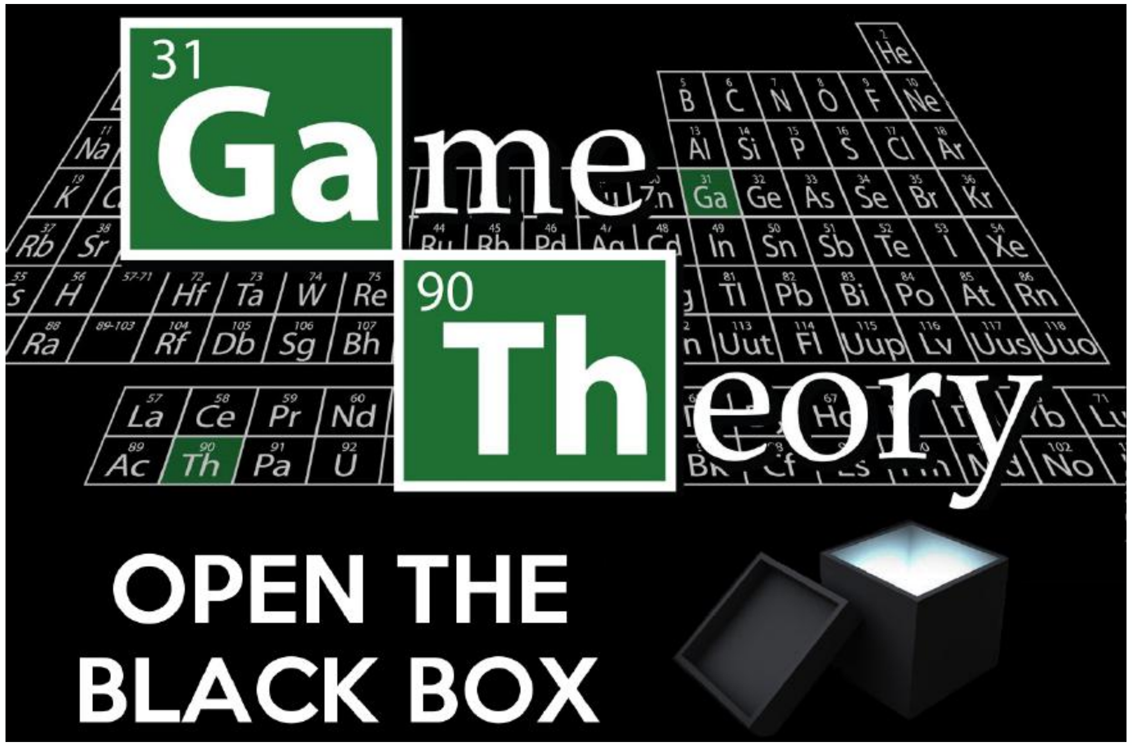
Determining the hyperplane can be viewed as a two-player game

- *one player trying to give the other the most challenging points to classify*
- *the other player is trying to find the best hyperplane*
- *the two players will converge to the eventual solution*

The method in which the player selects a hyper-plane is traditionally calculated via quadratic programming algorithms, but has also been achieved via iterative game theory and the chip-firing classifier

Bell, J. *Machine Learning: Hands-On for Developers and Technical Professionals; John Wiley & Sons, Inc.: Indianopolis, IN, 2014.*

Chemical Game Theory



- **Basics of Game Theory**
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- **Game Theory in Computer Science**
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- **Game Theory in Chemistry**
  - Case 1: deciding an optimal dft functional
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Evolutionary game theory

- similar to normal game theory, but the payoff is reproductive success and players don't need to act rationally

The hawk-dove game

	Dove	Hawk
Dove	(V/2,V/2)	(0,V)
Hawk	(V,0)	((V-C)/2, (V-C)/2)

V = Resources

C = Cost of Conflict

4 outcomes

Dominance – one player vanishes

Bistability – either player vanishes depending on the initial mixture

Coexistance – A & B exist in stable proportions

Neutrality – A & B only subject to random drift

evolutionary stable – a strategy that if almost every player of a species follows, no mutant can successfully invade

For a review, see: Nowak, M. A.; Sigmund, K. *Science* **2004**, *303*, 793.

Can get into significantly more complicated scenarios

iterated prisoners dilemma explains altriusm

	Screams	No scream
Screams	(-1,-1)	(-1,0)
No scream	(0,-1)	(-10, -10)

3 species can get into rock-paper-scissors types scenarios

Uta stansburiana Lizard

Coevolution

newt and gartner snake

- Mutation in virology
- Host–parasite interactions
- Development of language
- Sex-ratio theory
- Resource allocation
- Cancer cell-normal cell interactions
- Mate choice
- Sibling rivalry

… and more

Chemical Game Theory

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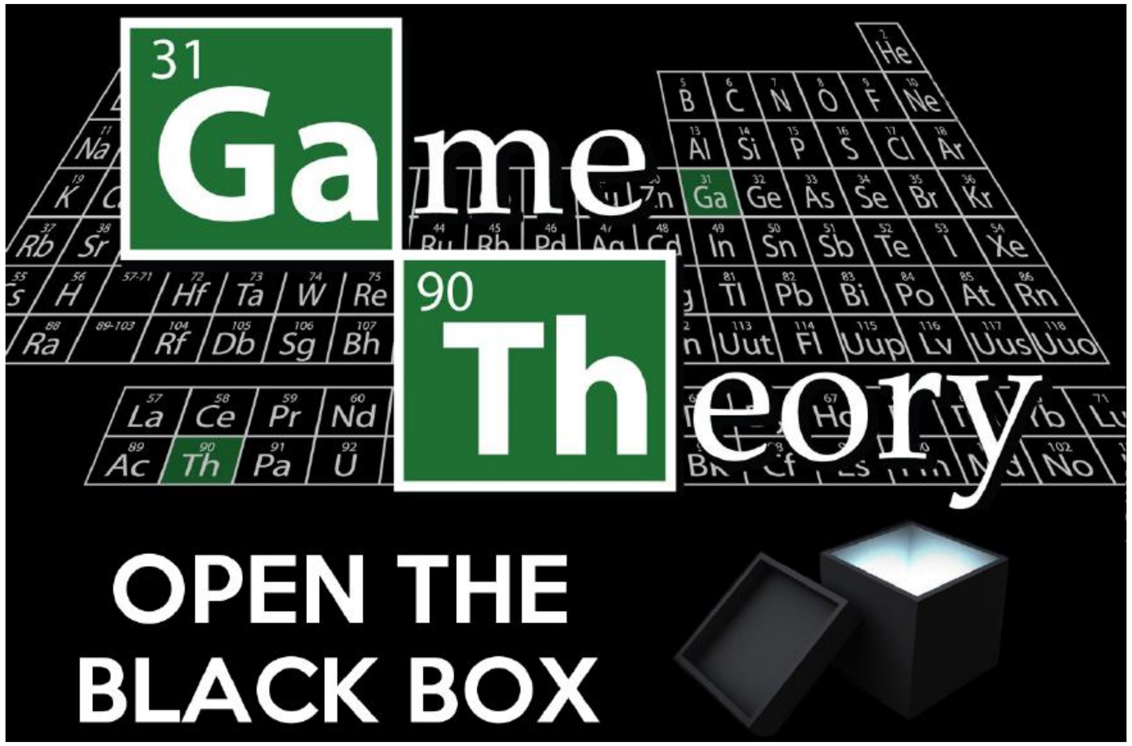
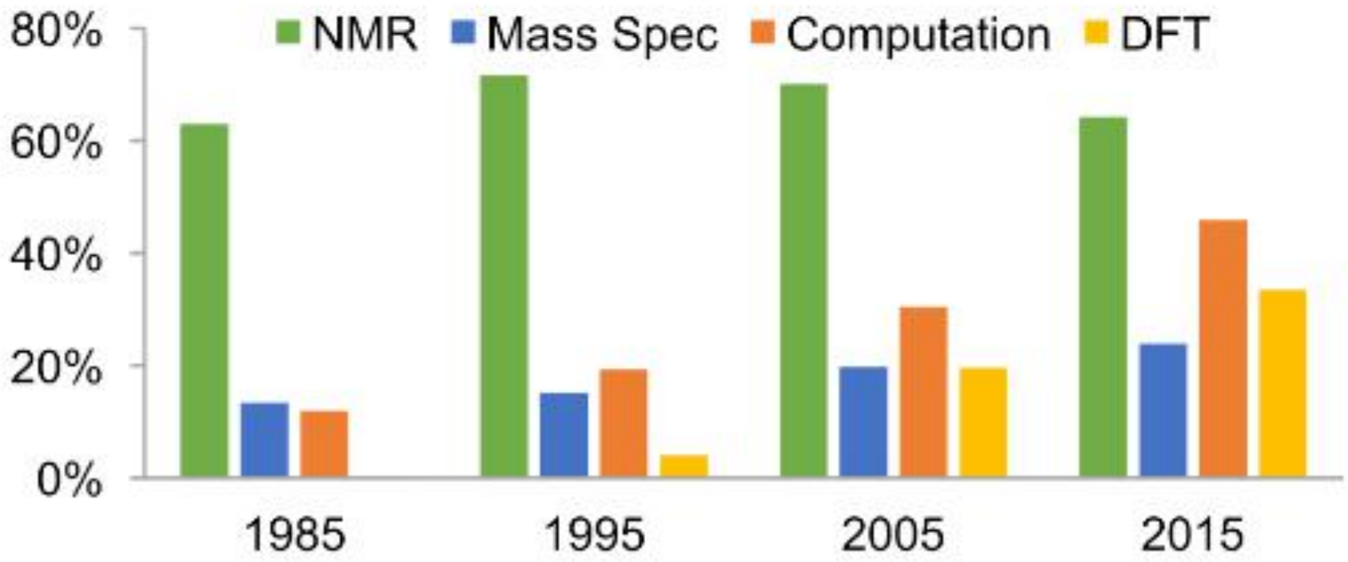


Selecting a proper dft functional

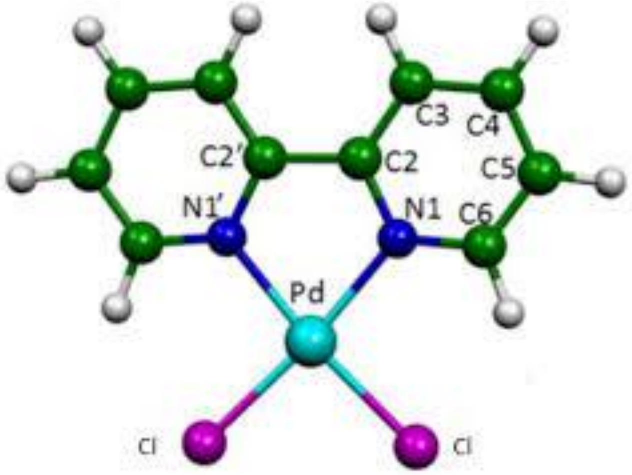
There are hundreds if not thousands of functionals with new types being customized for specialized problem types

Selecting a suitable functional and basis set can be challenging

Waller and coworkers developed Decider which relies upon game theory techniques to determine an optimal functional



percentage of ACS publications using the given tool



McAnanama-Brereton, S.; Waller, M. P. *J. Chem. Inf. Model.* **2018**, *58*, 61.

Selecting a proper dft functional

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3 players

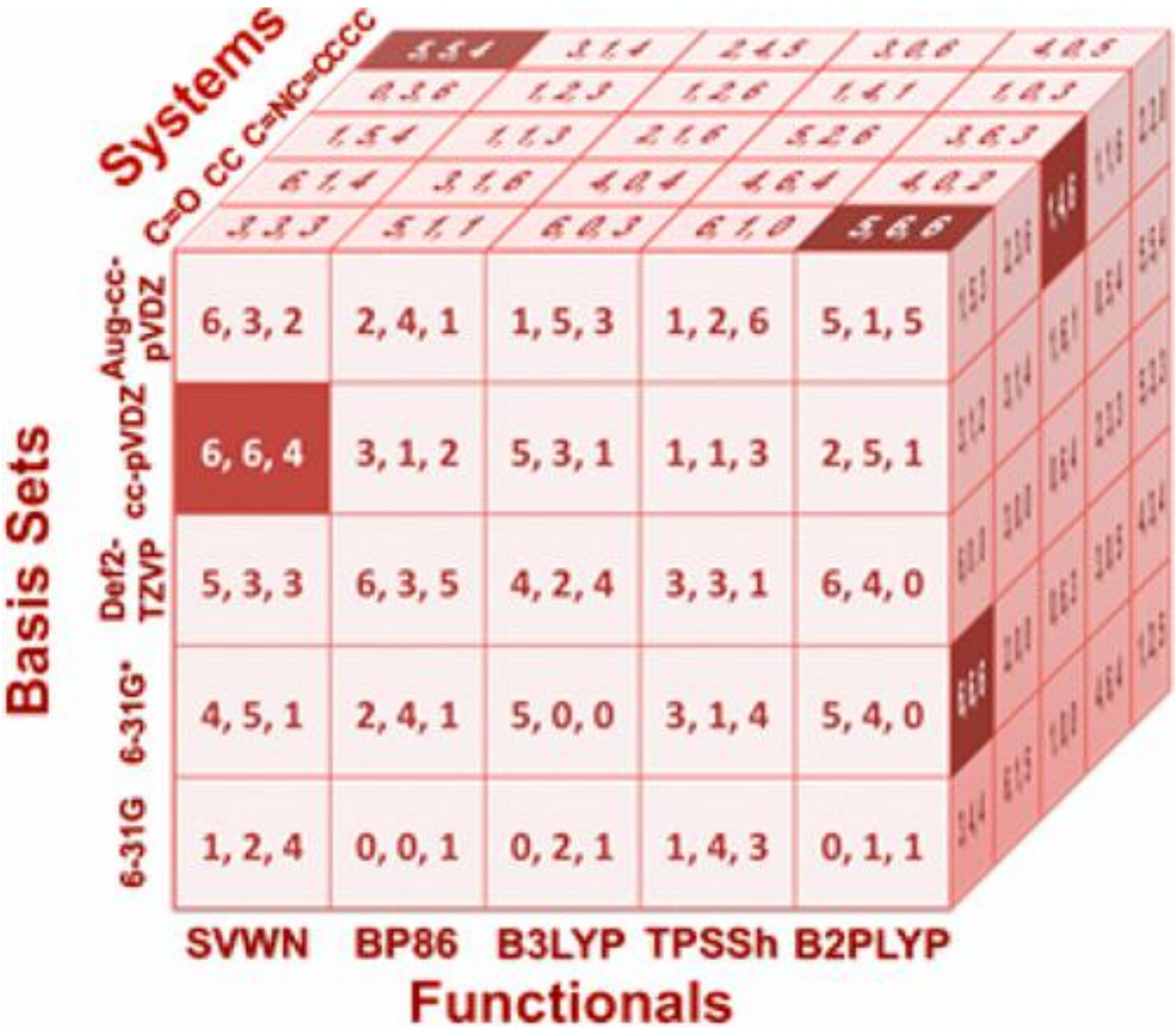
Accuracy – the performance of a basis set and functional relative to a reference set (mean absolute percent deviation or MAPD)

Complexity – the complexity of the basis set and functional relative to the complexity of the molecule being studied

Similarity – the similarity of the current query relative to a set of benchmark systems; measured as a Tanimoto score

McAnanama-Brereton, S.; Waller, M. P. *J. Chem. Inf. Model.* **2018**, *58*, 61.

Decider in action



Basis Sets	SVWN	BP86	B3LYP	TPSSh	B2PLYP
Aug-cc-pVDZ	6, 3, 2	2, 4, 1	1, 5, 3	1, 2, 6	5, 1, 5
cc-pVDZ	6, 6, 4	3, 1, 2	5, 3, 1	1, 1, 3	2, 5, 1
Def2-TZVP	5, 3, 3	6, 3, 5	4, 2, 4	3, 3, 1	6, 4, 0
6-31G*	4, 5, 1	2, 4, 1	5, 0, 0	3, 1, 4	5, 4, 0
6-31G	1, 2, 4	0, 0, 1	0, 2, 1	1, 4, 3	0, 1, 1

Created a 3-D matrix and then searched for Nash equilibria

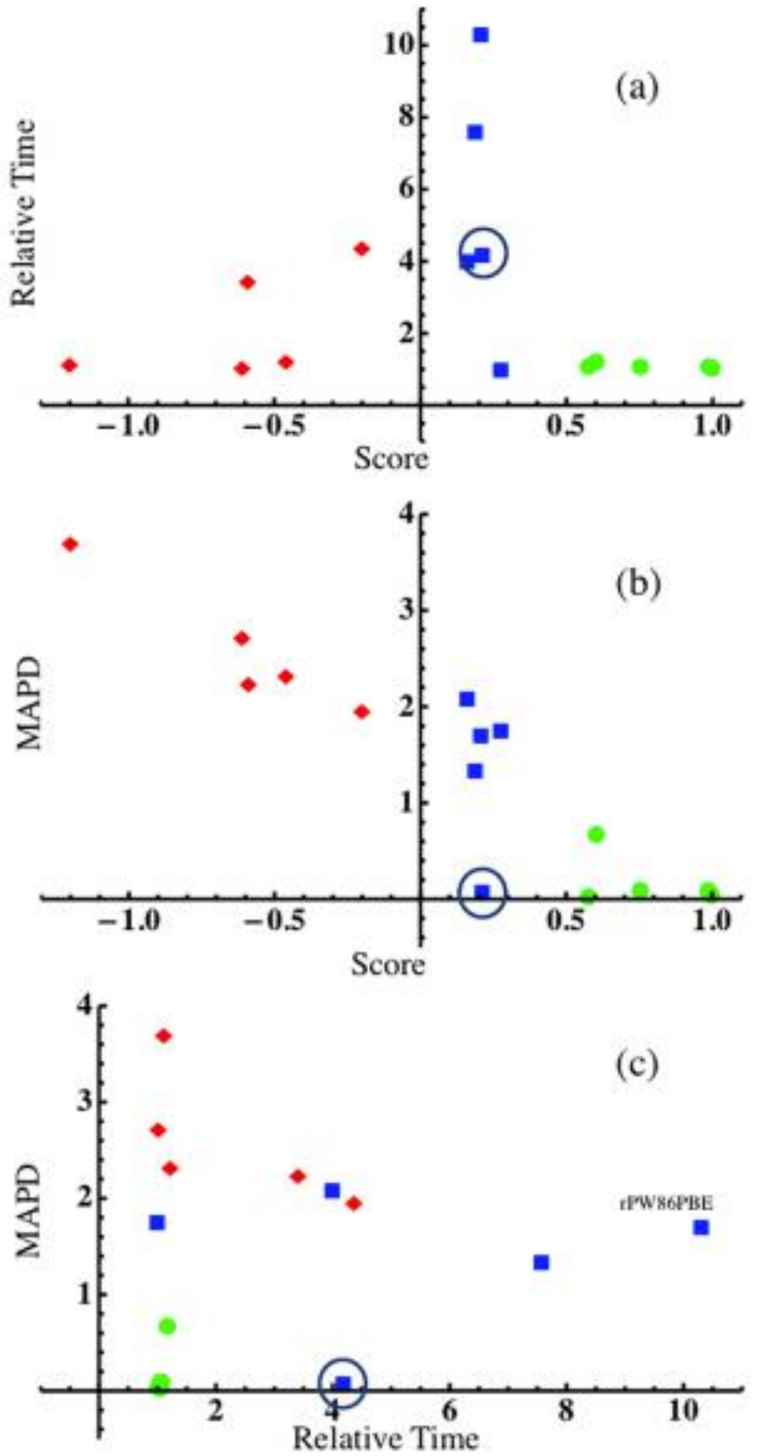
McAnanama-Brereton, S.; Waller, M. P. *J. Chem. Inf. Model.* **2018**, *58*, 61.

Decider in action

Tested the developed system on Hobza's S22 benchmarks

functional	basis set	score[a]
BLYP-D3	def2-QZVP	1
SVWN	def2-QZVP	0.988
PBE-D3	def2-QZVP	0.755
OPBE-D3	def2-QZVP	0.604
TPSS-D3	def2-QZVP	0.577
	● ● ●	
mPWLYP[34,35,75]	def2-QZVP	−0.2
BP86	def2-QZVP	−0.46
B97[76,77]	def2-QZVP	−0.59
BLYP	def2-QZVP	−0.61
OLYP[34,35,78]	def2-QZVP	−1.2

The top 5, middle 5, and bottom 5 functionals were then subjected to calculations in Gaussian and Orca



highest ●

middle ■

lowest ◆

McAnanama-Brereton, S.; Waller, M. P. *J. Chem. Inf. Model.* **2018**, *58*, 61.

Challenges of Exploring Novel Chemical Space

Estimated 10^{60} pharamacologically relevant small molecules

Discovering new technologies via conventional methods is time intensive – generally 15 to 20 years

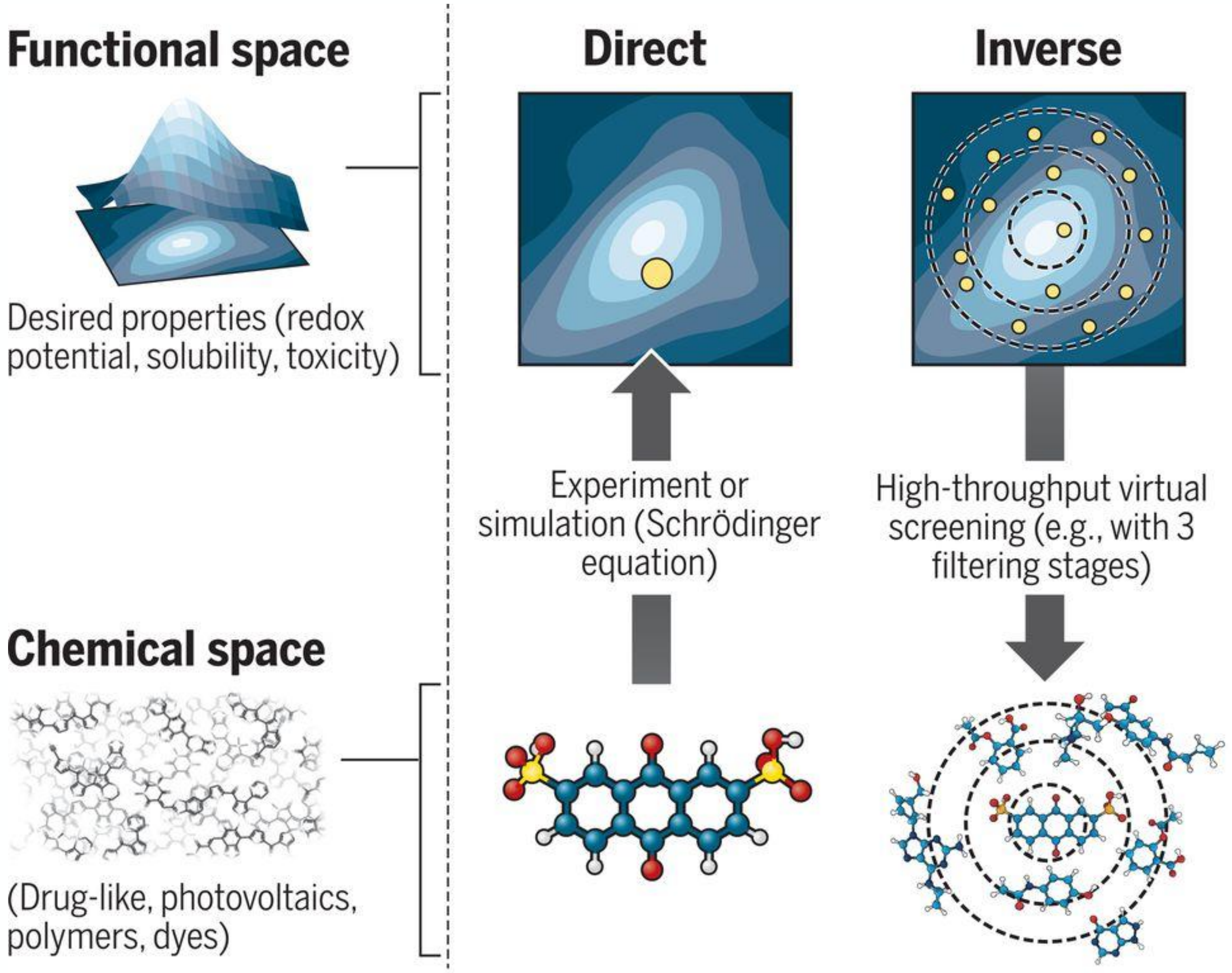
Until 2014, 49% of small molecule cancer drugs were natural products and their derivatives

Can we develop a method to more efficently explore chemical space and identify potential hits?

Inverse design starts from desired properties and ends in chemical space

Sanchez-Lengeling, B.; Aspuru-Guzik. A. *Science* **2018**, *361*, 360.

Direct vs inverse design in exploring chemical space



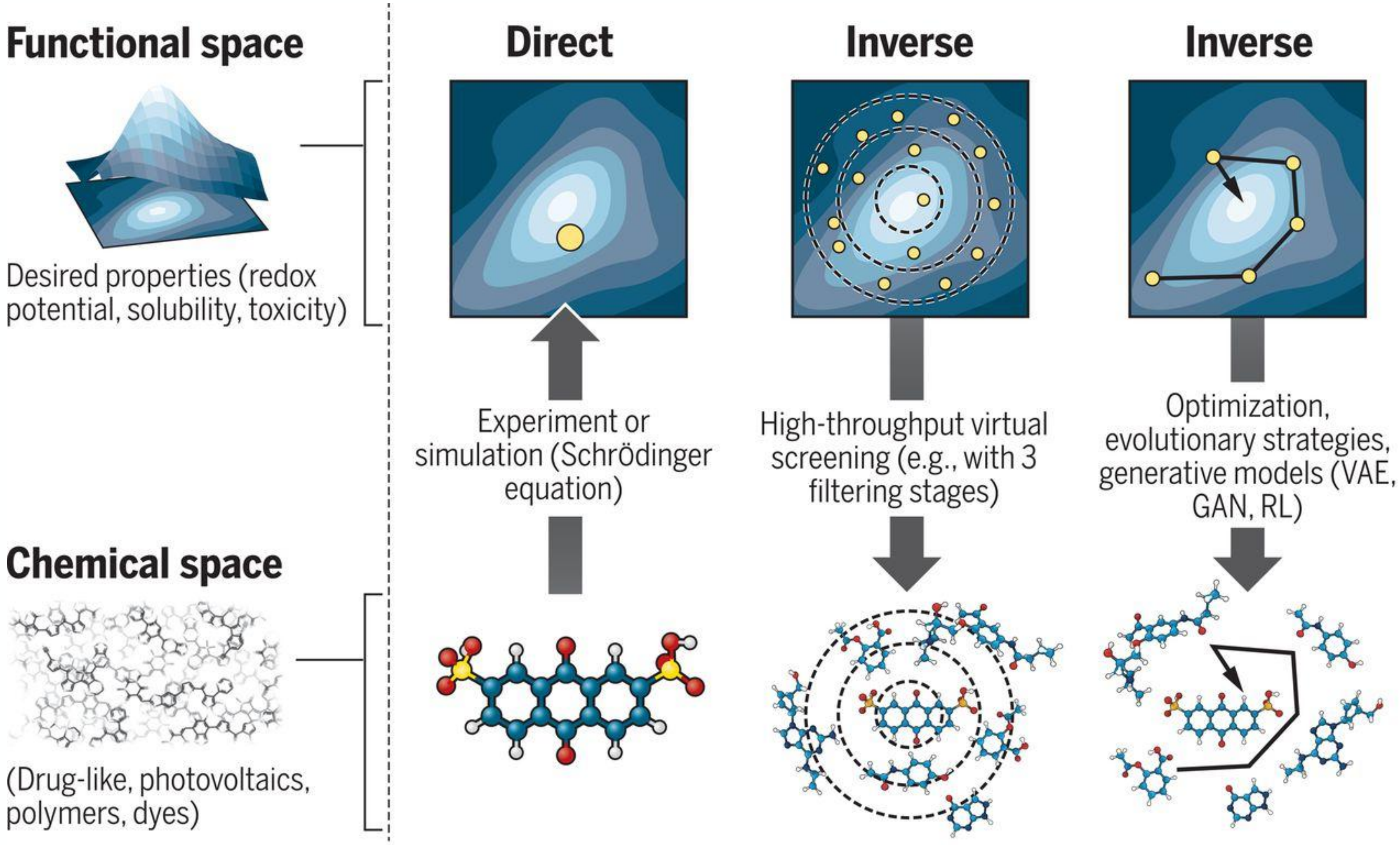
Direct design - Pick a specific compound and synthesize or simulate it

High Throughput Virtual Screening - Somewhat of a hybrid between inverse and direct design

- Starts with an initial set of molecules built on a researchers intuition
- Molecules are then narrowed down by being sorted through a range of filters

Sanchez-Lengeling, B.; Aspuru-Guzik. A. *Science* **2018**, *361*, 360.

Direct vs inverse design in exploring chemical space



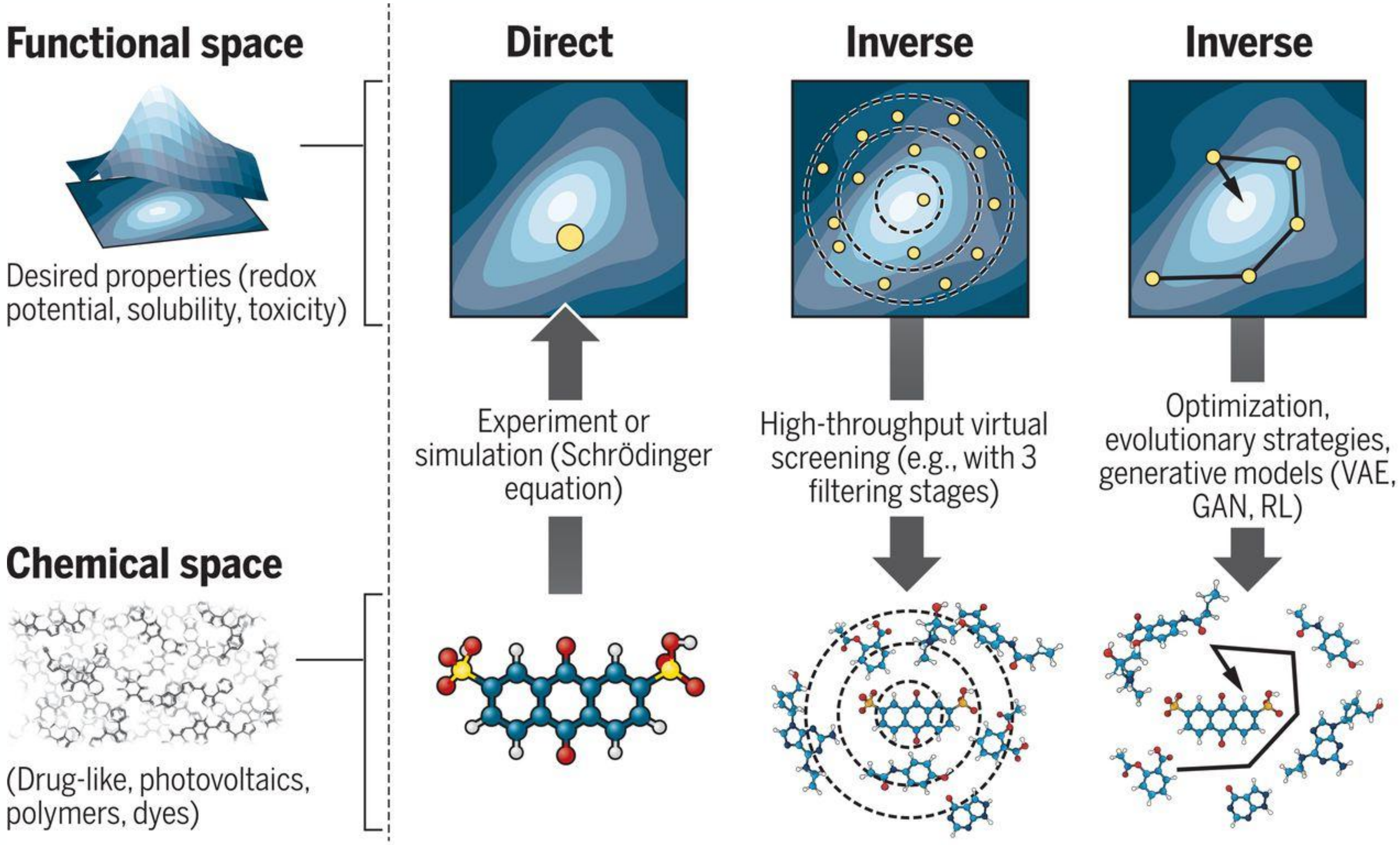
Pure Inverse Design

Evolution Strategy - A global optimization strategy that involves structured iterative searches

- parameter vectors (“genotypes”) are perturbed (“mutated”) and their objective funtional value (“fitness”) is evaluated

Sanchez-Lengeling, B.; Aspuru-Guzik. A. *Science* **2018**, *361*, 360.

Direct vs inverse design in exploring chemical space



Pure Inverse Design

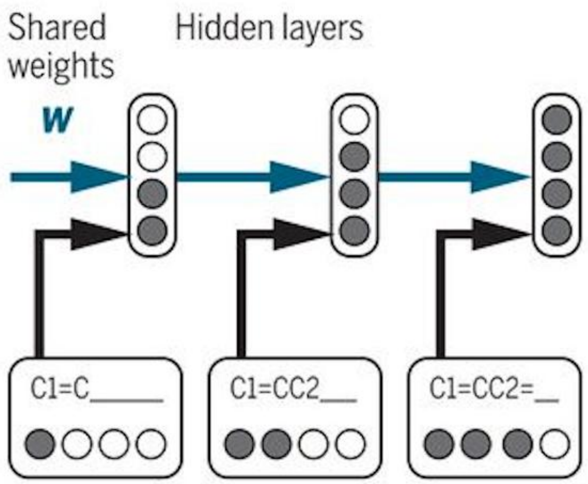
Generative Models - Attempts to determine a joint probability distribution *p*(x,y)- the probability of observing both the molecular representation and the desired property

- differs from a discriminative model which tries to determine a conditional probability *p(xly)* – the probability of observing properties y given molecule x

Sanchez-Lengeling, B.; Aspuru-Guzik. A. *Science* **2018**, *361*, 360.

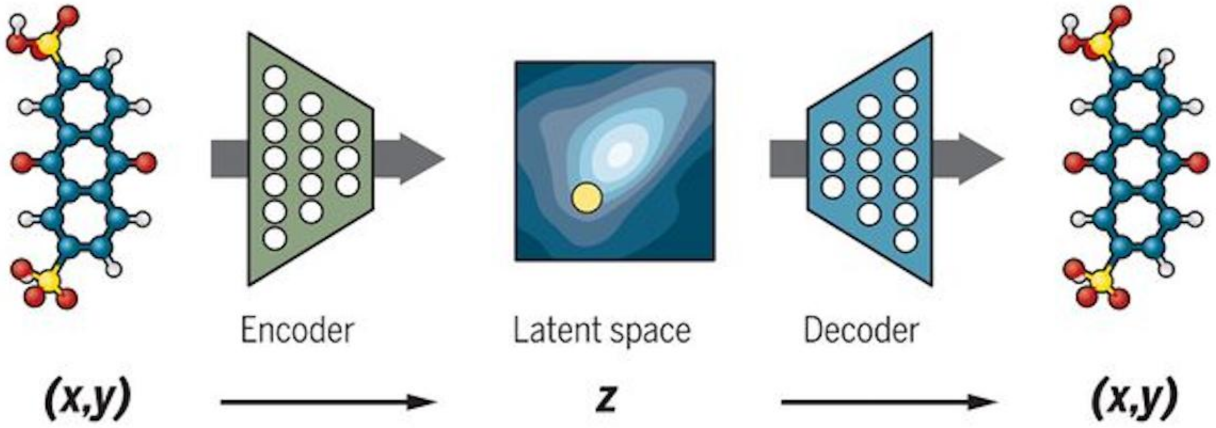
Types of generative models

Recurrent Neural Network (RNN)



- common starting point
- create sequences incrementally
- Long short-term memory (LSTM) allows RNN to take into account time-dependent patterns

Variational Autoencoders (VAE)



An encoder maps the molecule as a vector into a lower dimensional space, know as a latent space

A molecule is represented as a probability distribution over latent space

The VAE uses probability distributions to estimate the latent space

A decoder maps the latent space representation back to a molecule

Sanchez-Lengeling, B.; Aspuru-Guzik. A. *Science* **2018**, *361*, 360.

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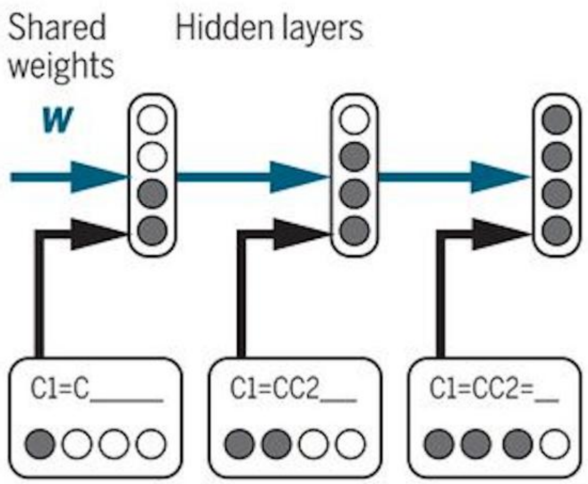

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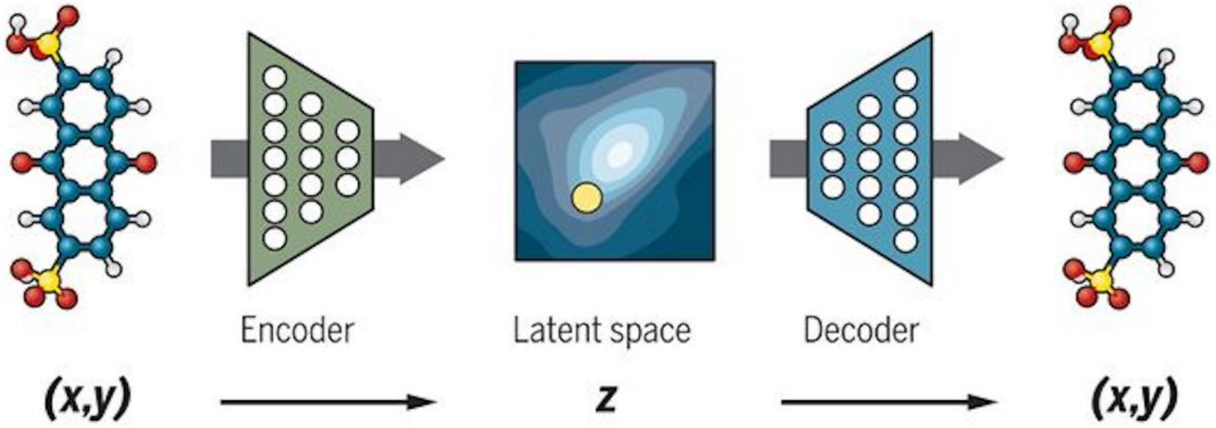
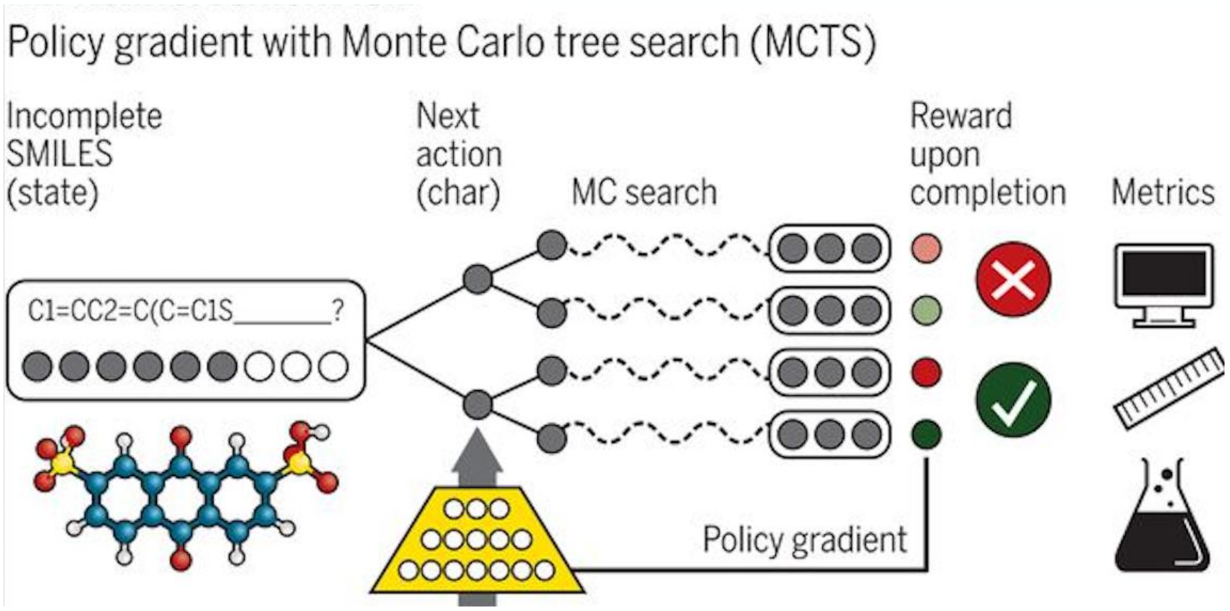


Reinforcement Learning (RL)



- an agent gives an output, which is then evaluated and returned to the agent so it can learn from it
- A generator must learn how to add smiles charactors to maximize some reward (property)
- As these properties can only be evaluated at the end, a Monte-Carlo tree search is generally used

Sanchez-Lengeling, B.; Aspuru-Guzik. A. *Science* **2018**, *361*, 360.

Types of generative models

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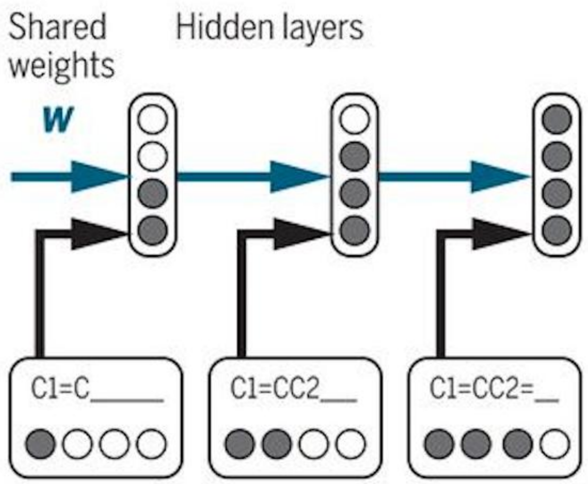

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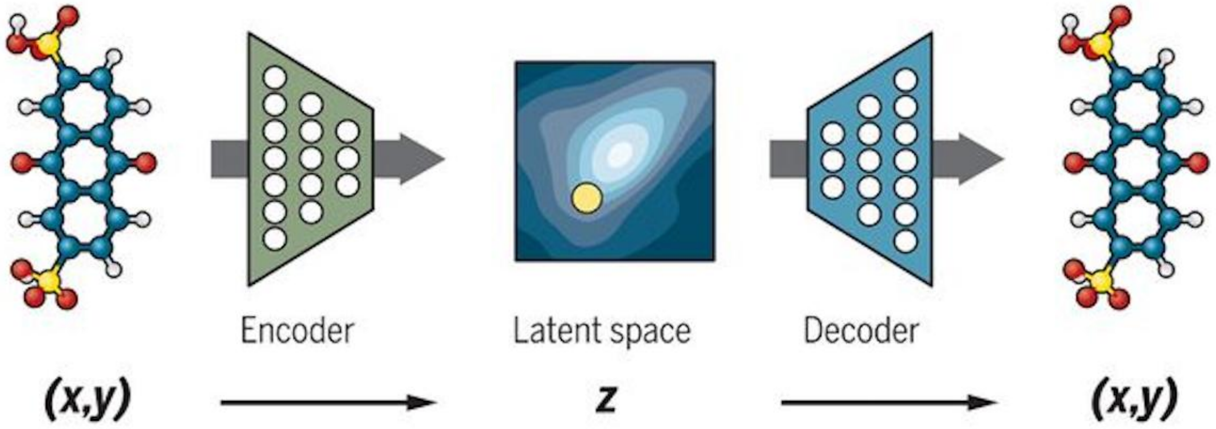

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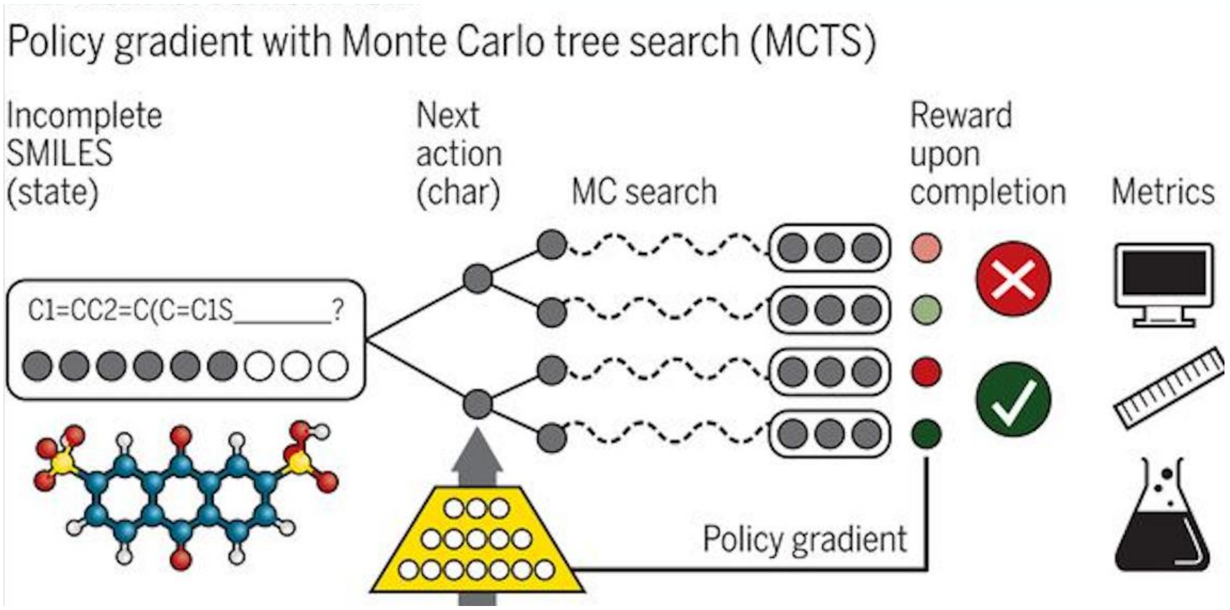
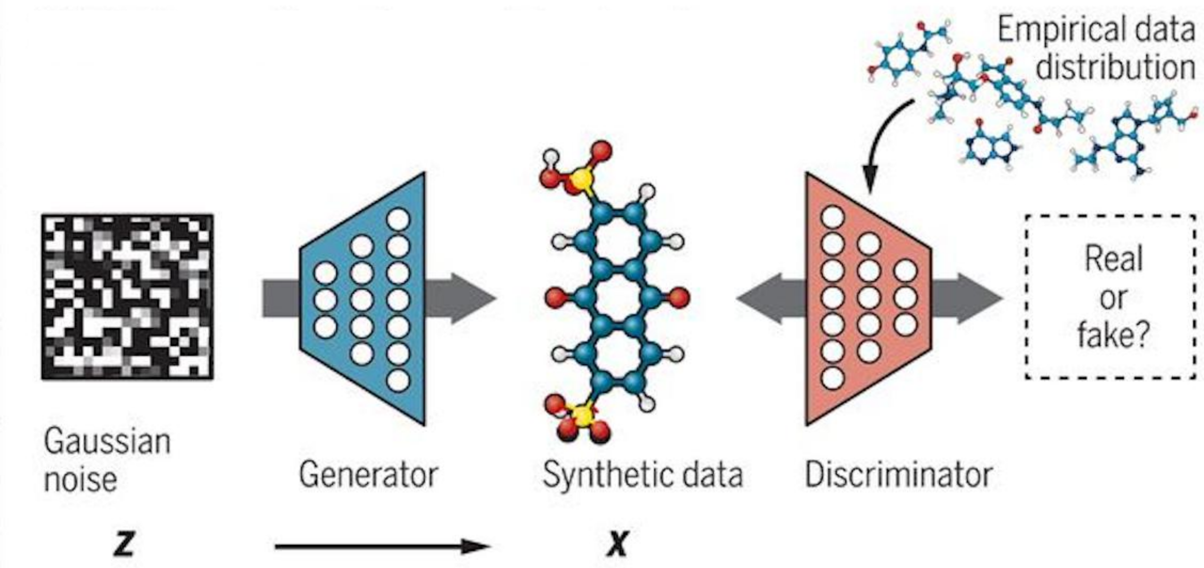


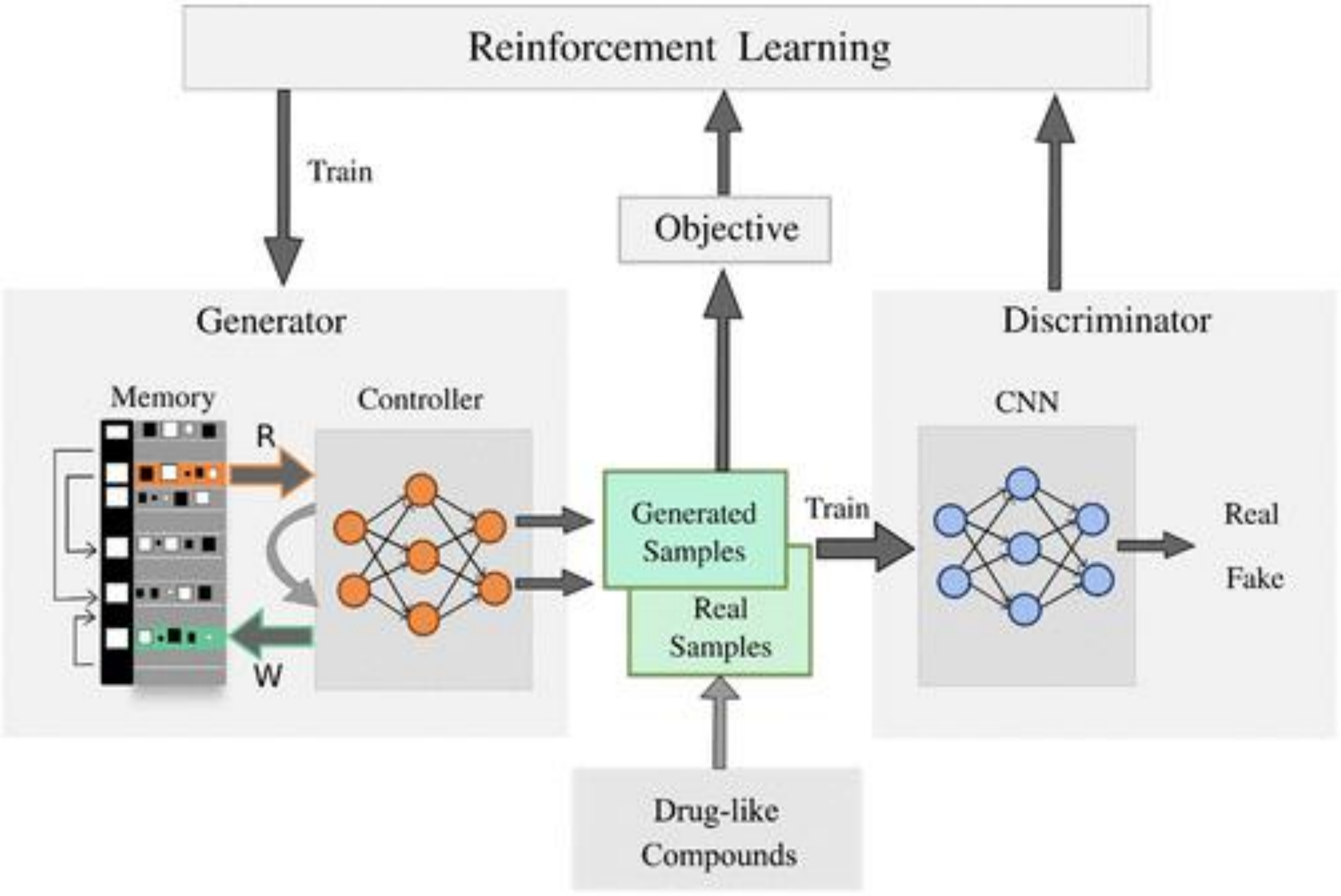
Generative Adversarial Networks (GANs)



Sanchez-Lengeling, B.; Aspuru-Guzik. A. *Science* **2018**, *361*, 360.

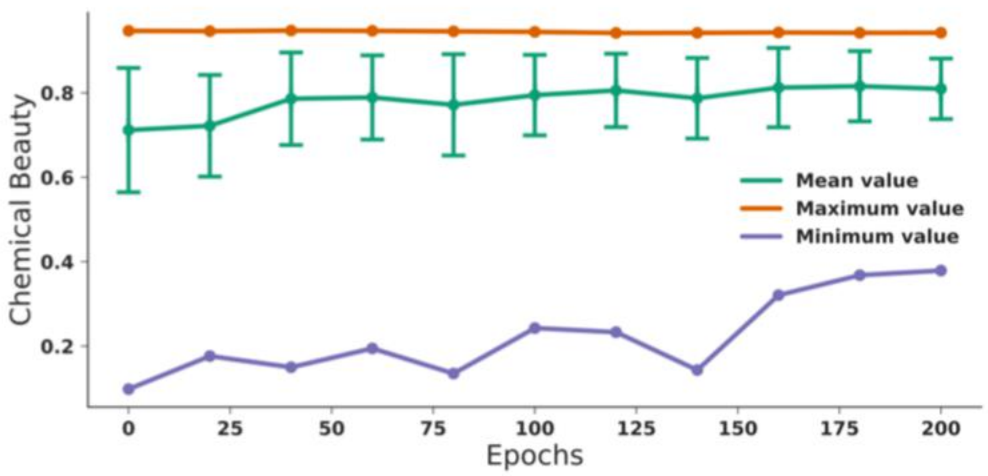
Applying generative models to pharmacologic systems

ORGANIC (Objective-Reinforced Generative Adversarial Network for Inverse-design Chemistry) and RANC (Reinforced Adversarial Neural Computer) both merge GANs and RL to achieve inverse design



Fed a subset of 15,000 drug-like compounds into the system

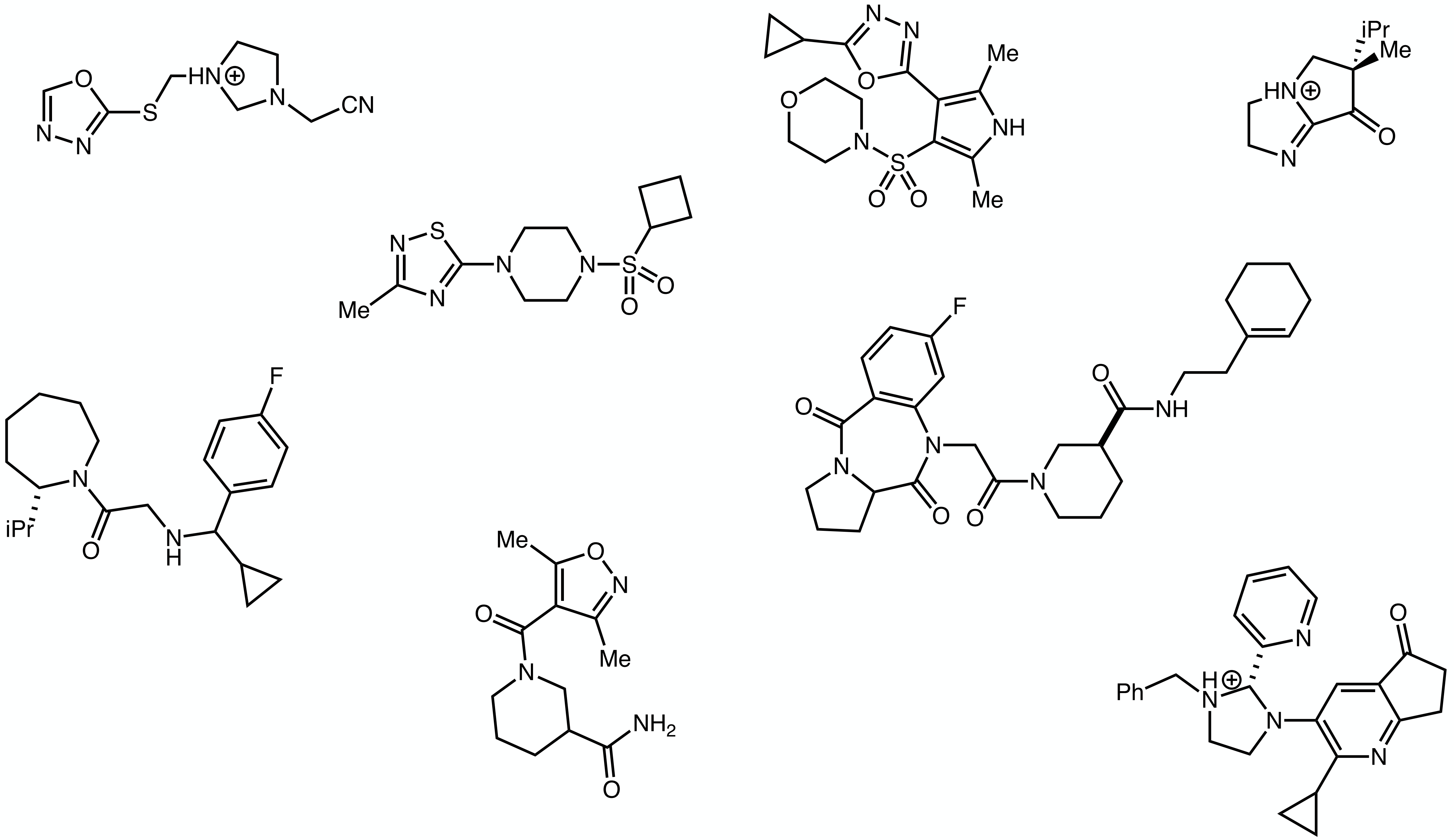
The system is then run through a number ~100 training epochs



Sanchez-Lengeling, B.; Aspuru-Guzik. A. *Science* **2018**, *361*, 360., Putin, E. et. al. *J. Chem. Inf. Model.* **2018**, *58*, 1194., https://doi.org/10.26434/chemrxiv.5309668.v3

Applying generative models to pharmacologic systems

Selected compounds generated by ORGANIC and RANC



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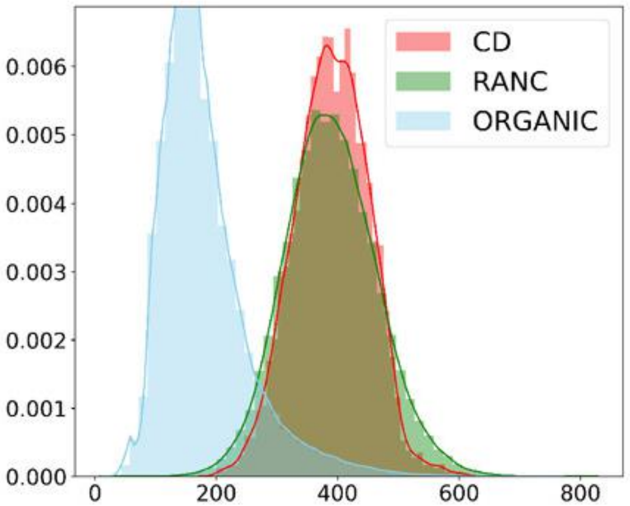
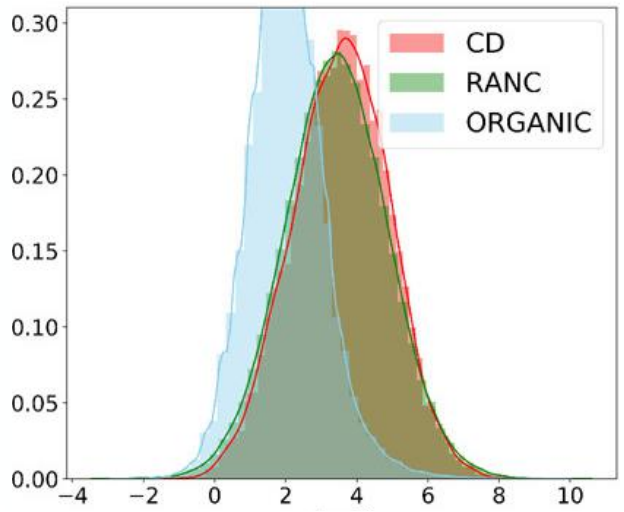
Applying generative models to pharmacologic systems

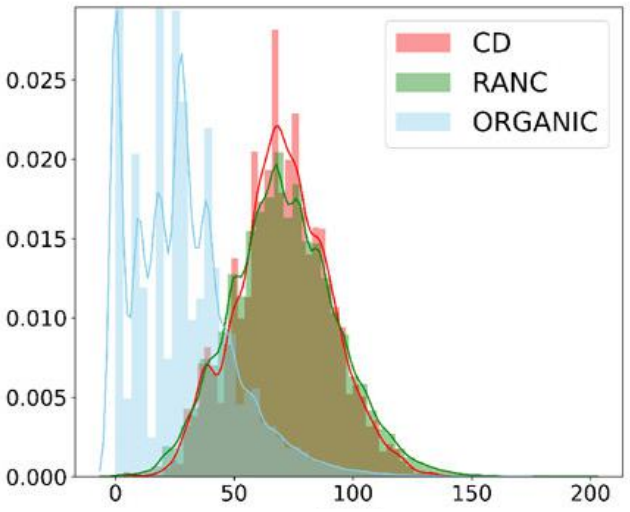
Comparing the performance of RANC and ORGANIC against the inital data



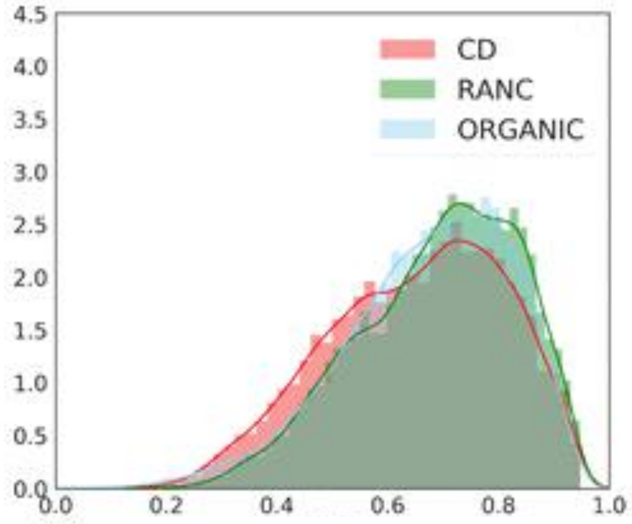
Molecular Weight



logP



TPSA



QED

	RANC	*ORGANIC*
avg. length	46	23
valid %	58	87
unique %	48	18

Sanchez-Lengeling, B.; Aspuru-Guzik. A. *Science* **2018**, *361*, 360., Putin, E. et. al. *J. Chem. Inf. Model.* **2018**, *58*, 1194., https://doi.org/10.26434/chemrxiv.5309668.v3

Inverse design forms a powerful platform

pharmacologic properties

organic photovoltaics

reaction synthesis planning

Inverse Design

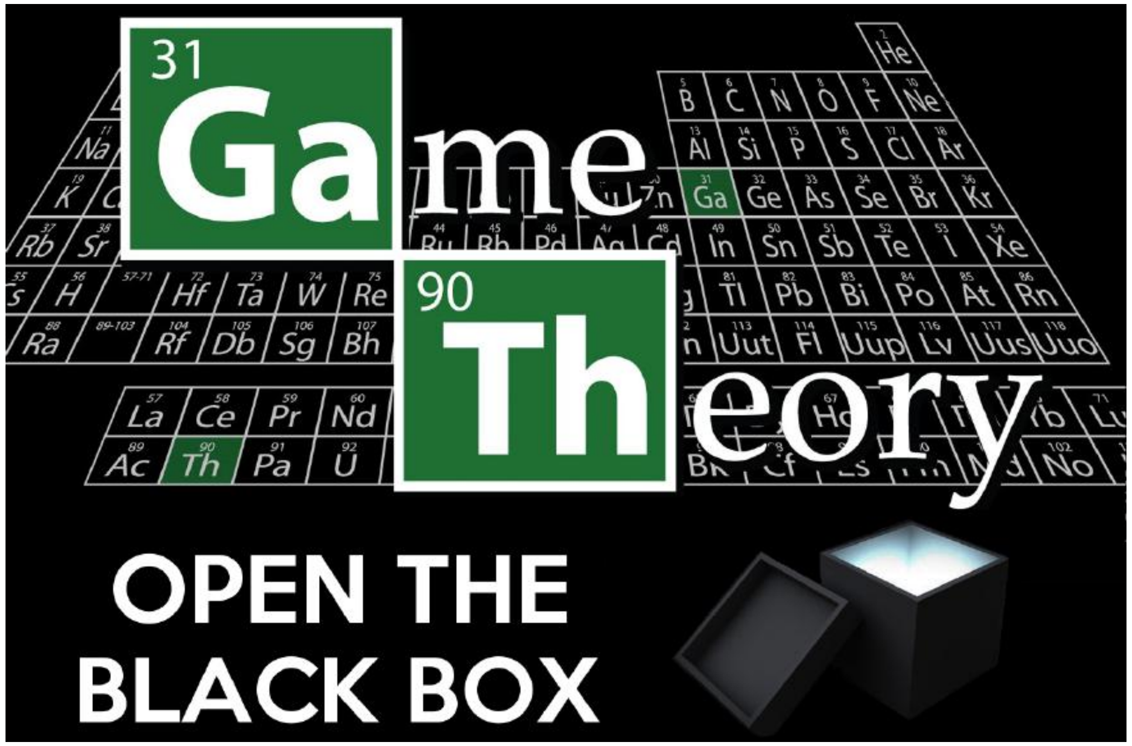
OLEDs

biological redox potentials

flow batteries

Chemical Game Theory

- **Basics of Game Theory**
    - Prisoners Dilemma
    - Battle of the Sexes
    - Rock Paper Scissors
    - Centipede Game
    - Iterated Prisoners Dilemma
- **Chemical Game Theory**
- **Game Theory in Computer Science**
- **Game Theory in Biology**
- **Game Theory in Chemistry**
    - Case 1: deciding an optimal dft functional
    - Case 2: inverse design



Questions?


WALLY, THESE PEER REVIEWS ARE LIKE THE FAMOUS "PRISONER'S DILEMMA."
IF YOU RAT ON ME BUT I SAY GOOD THINGS ABOUT YOU, YOU GET THE BIGGEST RAISE. BUT IF WE PRAISE EACH OTHER WE CAN BOTH GET A SMALL RAISE.
WALLY, IF YOU RAT HIM OUT, I'LL LET YOU LOOK AT MY "VICTORIA'S SECRET" CATALOG.
THIS IS EXACTLY WHY THERE ARE NO COED PRISONS.
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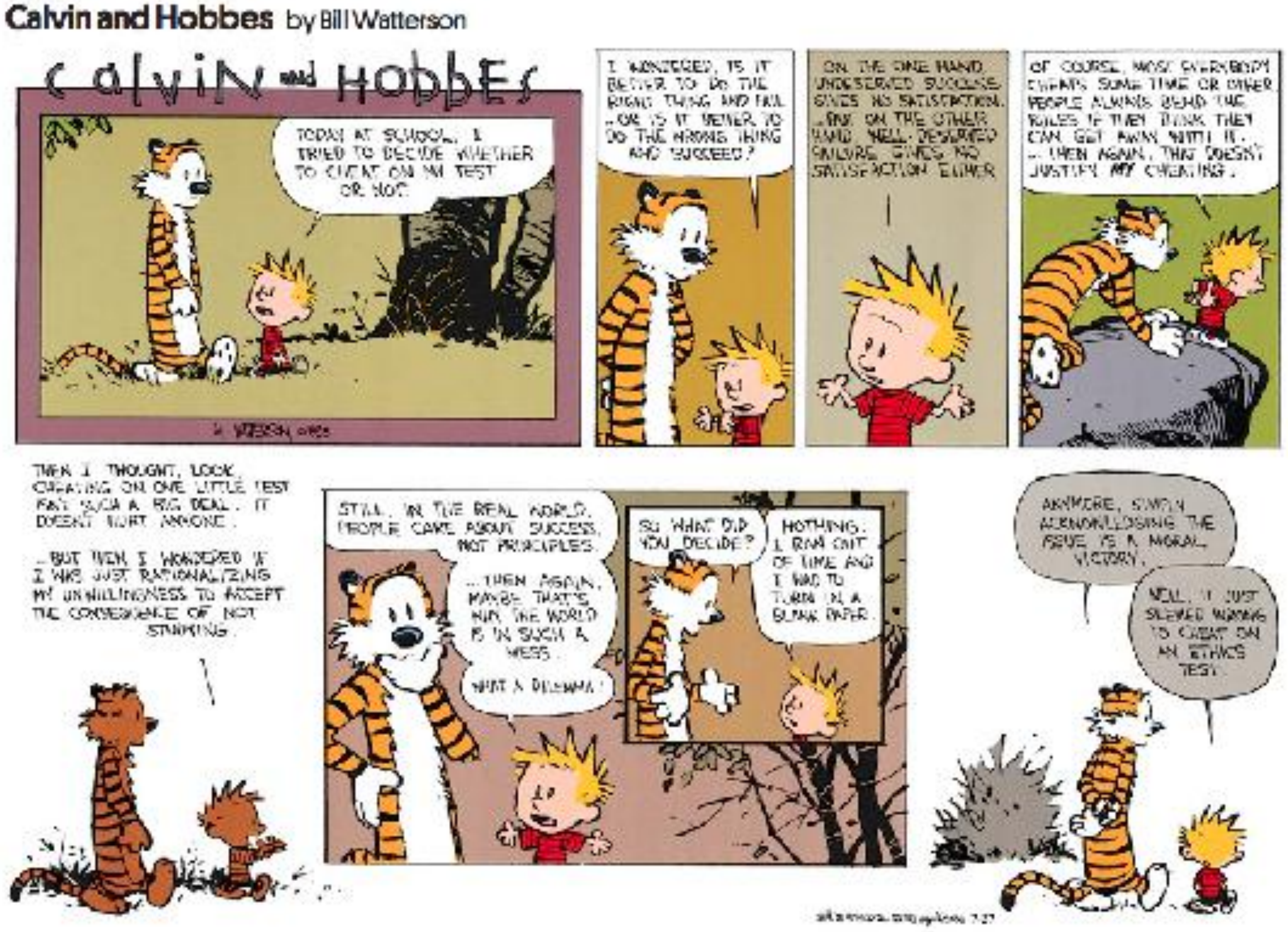
Calvin and Hobbes by Bill Watterson
CALVIN and HOBBES
TODAY AT SCHOOL, I TRIED TO DECIDE WHETHER TO CHEAT ON MY TEST OR NOT.
I WONDERED, IS IT BETTER TO DO THE RIGHT THING AND FAIL ...OR IS IT BETTER TO DO THE WRONG THING AND SUCCEED?
ON THE ONE HAND, UNDESERVED SUCCESS GIVES NO SATISFACTION. ...BUT ON THE OTHER HAND, WELL-DESERVED FAILURE GIVES NO SATISFACTION EITHER.
OF COURSE, MOST EVERYBODY CHEATS SOME TIME OR OTHER. PEOPLE ALWAYS BEND THE RULES IF THEY THINK THEY CAN GET AWAY WITH IT. ...THEN AGAIN, THAT DOESN'T JUSTIFY MY CHEATING.
THEN I THOUGHT, LOOK, CHEATING ON ONE LITTLE TEST ISN'T SUCH A BIG DEAL. IT DOESN'T HURT ANYONE.
...BUT THEN I WONDERED IF I WAS JUST RATIONALIZING MY UNWILLINGNESS TO ACCEPT THE CONSEQUENCE OF NOT STUDYING.
STILL, IN THE REAL WORLD, PEOPLE CARE ABOUT SUCCESS, NOT PRINCIPLES.
...THEN AGAIN, MAYBE THAT'S WHY THE WORLD IS IN SUCH A MESS.
WHAT A DILEMMA!
SO WHAT DID YOU DECIDE?
NOTHING. I RAN OUT OF TIME AND I HAD TO TURN IN A BLANK PAPER.
ANYMORE, SIMPLY ACKNOWLEDGING THE ISSUE IS A MORAL VICTORY.
WELL, IT JUST SEEMED WRONG TO CHEAT ON AN ETHICS TEST.